



## EXAMINATION OF PROPER RANDOMNESS OF NUMBERS OF M. G. KENDALL AND B. BABINGTON SMITH'S RANDOM NUMBERS TABLE: RUN TEST

**Brajendra Kanta Sarmah\* & Dr. Dhritikesh Chakrabarty\*\***

\* Department of Statistics, Bholanath College, Dhubri, Assam

\*\* Department of Statistics Handique Girls' College, Guwahati, Assam

### **Abstract:**

*To Examine whether the numbers generated by M.G. Kendall and B. Babington Smith are properly random or not one may apply nonparametric tests for randomness. Various types of nonparametric tests are ordinary run test, Wald – wolfowitz Run test, Sign test, Mann-whitney – Wilcoxon U-test, Mann-Kendall test. Here, we shall apply ordinary run test by taking the median of the number to test the proper randomness of the numbers of Kendall and Smith random members table. It is observed that Kendall and Smith's random numbers table may be regarded random with respect to run test.*

**Key Words:** Kendall and Smith Random Number Table, Testing of Randomness, Non Parametric Test & Run Test.

### **Introduction:**

A run is defined as a series of increasing values or a series of decreasing values. The number of increasing or decreasing values is the length of the run. In a random number data set, the probability that a particular number is larger or smaller than the median value follows a binomial distribution, which forms a basis of the run test.

Drawing of random sample has been found necessary in most of the investigations. The most practical method of selecting a random sample consists of the use of random numbers table. Commonly used random numbers tables are LHC Tippett, Fisher and Yates, Kendall and Babington Smith and Rand Corporation. These tables have been subjected to various statistical tests of randomness. Proper randomness of Kendall and Smith Random Numbers table has been examined by B.K. Sarmah and D. Chakrabarty (Feb, 2015 IJESRT) by applying Chi-square test for testing the significance of difference between observed frequency of each of the digit in the table and corresponding theoretical ( expected ) frequency. The randomness of the digits of Kendall and Smith random numbers table are also examined by applying t-test (July-Dec/15 ABJMI) for amount of deviation of the observed number of occurrence and theoretical (expected) number of occurrence of the respective digits. Although Chi-square test shows randomness, t-test shows significant deviation from proper randomness.

In this study an attempt has been made to test the proper randomness of number of Kendall and Smith table by applying run test.

In statistical literature “proper randomness” refers to a process that can produce independently and identically distributed (iid) samples (number) having equal probability of occurrences.

### **Materials and Method:**

Although Kendall and Smith random number table consists of 25000 four digit random numbers, viz 0001, 0002, 0003 ..... 9999, here we take a sample of 15,000 random numbers to test the proper randomness.

In order to test the randomness of the occurrence of the numbers we note that if the occurrence is random then the theoretical median of the observations will be  $4999.5 \left[ \frac{4999+5000}{2} \right]$

Let us consider a sequence of runs from the table by writing A if the observation is above median and B if the observation is below the median.

Let us consider the following hypothesis

$H_0$  : The occurrence of numbers in the table is in random manner.

$H_1$ : The occurrence of the numbers in the table is not in random manner.

Under the null hypothesis  $H_0$ , the test statistic is given by.

$$Z = \frac{U - E(U)}{\sqrt{V(U)}} \sim N(0,1)$$

Where,

$U$  = Number of observed runs

$$E(U) = \frac{n+2}{2}$$

$$V(U) = \frac{n}{4} \left( \frac{n-2}{n-1} \right)$$

$n$  = Number of observations considered.

We may accept  $H_0$ , iff  $|Z| < 2.58$  (1% probability level)

#### **Steps in the Method:**

In order to test the proper randomness of the numbers of Kendall and Smith table one is required to proceed with the following steps:

Step -1: In the first step runs of A and B are observed for first 200 numbers second 200 numbers ..... up to 75<sup>th</sup> 200 numbers.

Step -2: In the second step count the total number of runs for first 200, second 200..... upto 75<sup>th</sup> 200 numbers.

Step -3: In this step the values of  $Z$  are calculated by applying the formula

$$Z = \frac{U - E(U)}{\sqrt{V(U)}} \sim N(0, 1)$$

Step - 4: compare the calculated value of  $|Z|$  with 1.96 (at 5% Level of significance), or with 2.58 (at 1% level of significance)

Step-5: Draw conclusion as per the result obtained in step 4.

#### **Conclusion:**

From the table prepared for pattern of runs, number of runs and value of  $|Z|$ , it is observed that values of  $|Z|$  calculated for first 200, 2<sup>nd</sup> 200..... to last 75<sup>th</sup> 200 numbers are found to be insignificant comparing them with standard value of  $Z$  (2.58. at 1% level of significance)

Hence after applying run test it may be concluded that the numbers of M.G. Kendall and B. Babington Smith's table are properly random

**Table: Observed runs, number of runs and the value of  $|Z|$  statistic from M.G. Kendall and B. Babington Smith**

| Random Numbers Considered | 'Pattern of Runs           | No. of runs $U$ | $E(U) = \frac{n+2}{2}$ | $V(U) = \frac{n}{4} \left( \frac{n-2}{n-1} \right)$ | $ Z $ |
|---------------------------|----------------------------|-----------------|------------------------|-----------------------------------------------------|-------|
| 1 <sup>st</sup> 200       | ABABBAABBABAABBBABA.....   | 116             | 101                    | 49.75                                               | 2.13  |
| 2 <sup>nd</sup> 200       | ABBAABAABBAABBBABBB.....   | 88              | 101                    | 49.75                                               | 1.84  |
| 3 <sup>rd</sup> 200       | BBBABBABBBBBBAAAAABBB..... | 104             | 101                    | 49.75                                               | 0.43  |
| 4 <sup>th</sup> 200       | BABBABAABAAABBBABAAB....   | 109             | 101                    | 49.75                                               | 1.13  |
| 5 <sup>th</sup> 200       | ABBBAAABBBABBBBBAB.....    | 98              | 101                    | 49.75                                               | 0.43  |
| 6 <sup>th</sup> 200       | AAABAAABBAABBBABB.....     | 90              | 101                    | 49.75                                               | 1.56  |
| 7 <sup>th</sup> 200       | BBBBABBBAAABBBAB.....      | 96              | 101                    | 49.75                                               | 0.71  |
| 8 <sup>th</sup> 200       | AAAAABBBAAABBAABBBAB.....  | 97              | 101                    | 49.75                                               | 0.57  |
| 9 <sup>th</sup> 200       | AAABAAABABBAABBB.....      | 98              | 101                    | 49.75                                               | 0.43  |
| 10 <sup>th</sup> 200      | AAABBABBAABBBAAA.....      | 89              | 101                    | 49.75                                               | 1.70  |

|                      |                             |     |     |       |      |
|----------------------|-----------------------------|-----|-----|-------|------|
| 11 <sup>th</sup> 200 | AAAABABABBBBABBA.....       | 101 | 101 | 49.75 | 0    |
| 12 <sup>th</sup> 200 | ABAAABAABBBABABAAB.....     | 103 | 101 | 49.75 | 0.28 |
| 13 <sup>th</sup> 200 | BBAAABBBABBBABBA.....       | 95  | 101 | 49.75 | 0.85 |
| 14 <sup>th</sup> 200 | BBABABBBABBBB.....          | 113 | 101 | 49.75 | 1.70 |
| 15 <sup>th</sup> 200 | BABBBBAAABBBABBA.....       | 91  | 101 | 49.75 | 1.42 |
| 16 <sup>th</sup> 200 | BBAABABABAAAABABBB.....     | 111 | 101 | 49.75 | 1.42 |
| 17 <sup>th</sup> 200 | BBBBBABAABBBABAB.....       | 111 | 101 | 49.75 | 1.42 |
| 18 <sup>th</sup> 200 | BBBBAABBAABAAA.....         | 99  | 101 | 49.75 | 0.28 |
| 19 <sup>th</sup> 200 | AAAABBABBBABBA.....         | 92  | 101 | 49.75 | 1.28 |
| 20 <sup>th</sup> 200 | BBBAABBAABBBABBB.....       | 100 | 101 | 49.75 | 0.14 |
| 21 <sup>st</sup> 200 | AAABBABAAAABA.....          | 105 | 101 | 49.75 | 0.57 |
| 22 <sup>nd</sup> 200 | BBBBBABAABBA.....           | 105 | 101 | 49.75 | 0.57 |
| 23 <sup>rd</sup> 200 | BAABBBBABABBB.....          | 98  | 101 | 49.75 | 0.43 |
| 24 <sup>th</sup> 200 | BBABAABBAABBA.....          | 110 | 101 | 49.75 | 1.28 |
| 25 <sup>th</sup> 200 | ABABABABAAAABBB.....        | 102 | 101 | 49.75 | 0.14 |
| 26 <sup>th</sup> 200 | BAABBBABABABABABABAB..      | 118 | 101 | 49.75 | 2.41 |
| 27 <sup>th</sup> 200 | ABBBBABBBABBBBAABBB....     | 88  | 101 | 49.75 | 1.84 |
| 28 <sup>th</sup> 200 | BAABABBAABBBBABAABBA...     | 102 | 101 | 49.75 | 0.14 |
| 29 <sup>th</sup> 200 | BAAABBBBBAABABABA.....      | 96  | 101 | 49.75 | 0.71 |
| 30 <sup>th</sup> 200 | AAAAABBAABABABBBABABAA.     | 109 | 101 | 49.75 | 1.13 |
| 31 <sup>st</sup> 200 | BBBAAAABAABBBBAABBA... ..   | 97  | 101 | 49.75 | 0.57 |
| 32 <sup>nd</sup> 200 | BABABABBBABBBABBAABBA..     | 101 | 101 | 49.75 | 0    |
| 33 <sup>rd</sup> 200 | BABBABAABBBABBAABABAA.....  | 115 | 101 | 49.75 | 1.99 |
| 34 <sup>th</sup> 200 | AABBABBBBABAABBBBA.....     | 106 | 101 | 49.75 | 2.13 |
| 35 <sup>th</sup> 200 | ABAABABBBBAABAAAABBB.....   | 90  | 101 | 49.75 | 1.56 |
| 36 <sup>th</sup> 200 | BABABAAABBBABABABAAABAA.    | 116 | 101 | 49.75 | 2.13 |
| 37 <sup>th</sup> 200 | BABBABABBAABBBABBAABBA..    | 98  | 101 | 49.75 | 0.43 |
| 38 <sup>th</sup> 200 | AAAAABABBBBBAABABAA.....    | 103 | 101 | 49.75 | 0.28 |
| 39 <sup>th</sup> 200 | BBBBBAAABBBABBAABBA.....    | 105 | 101 | 49.75 | 0.56 |
| 40 <sup>th</sup> 200 | AABAAAABABAABABBB.....      | 111 | 101 | 49.75 | 1.42 |
| 41 <sup>st</sup> 200 | ABAAAAABBBABABAAAABBBBA..   | 114 | 101 | 49.75 | 1.84 |
| 42 <sup>nd</sup> 200 | ABBBAAABAABBBABABABBB...    | 90  | 101 | 49.75 | 1.56 |
| 43 <sup>rd</sup> 200 | BBABBABAAABABBABABAB....    | 108 | 101 | 49.75 | 0.99 |
| 44 <sup>th</sup> 200 | ABAAABBBBBAABBAABAA.....    | 99  | 101 | 49.75 | 0.28 |
| 45 <sup>th</sup> 200 | BBBBAAAABBAABBAABBB....     | 117 | 101 | 49.75 | 2.27 |
| 46 <sup>th</sup> 200 | BBBBAAAABBAABBAABBBBA...    | 85  | 101 | 49.75 | 2.27 |
| 47 <sup>th</sup> 200 | BBABAABBAABBBBAABABBA..     | 104 | 101 | 49.75 | 0.43 |
| 48 <sup>th</sup> 200 | ABAAABBBBABABBABBAAB...     | 116 | 101 | 49.75 | 2.13 |
| 49 <sup>th</sup> 200 | AABABBAABBBABBBBA.....      | 88  | 101 | 49.75 | 1.84 |
| 50 <sup>th</sup> 200 | BABBAABAABBBABBAABBAABBB... | 97  | 101 | 49.75 | 0.57 |
| 51 <sup>st</sup> 200 | AAAABBBBBAABABABBAABAA...   | 109 | 101 | 49.75 | 1.13 |
| 52 <sup>nd</sup> 200 | AABAABBBBABAABABBBBA...     | 95  | 101 | 49.75 | 0.85 |
| 53 <sup>rd</sup> 200 | ABABBAABBAABBBABBBBA...     | 100 | 101 | 49.75 | 0.14 |
| 54 <sup>th</sup> 200 | ABABBBABBBAAABABAAAB....    | 89  | 101 | 49.75 | 1.70 |
| 55 <sup>th</sup> 200 | BABBABABBBABBBABBBBA...     | 117 | 101 | 49.75 | 2.27 |
| 56 <sup>th</sup> 200 | BAABAABAABAABAAABAB....     | 110 | 101 | 49.75 | 1.28 |
| 57 <sup>th</sup> 200 | ABABBBABAABABBBBABAAA...    | 103 | 101 | 49.75 | 0.28 |
| 58 <sup>th</sup> 200 | BBAABBAABBBABBAABBBBA...    | 96  | 101 | 49.75 | 0.71 |
| 59 <sup>th</sup> 200 | BBBAAABBBAAABABABAAAAB...   | 105 | 101 | 49.75 | 0.57 |
| 60 <sup>th</sup> 200 | ABBBABBAABBBBBAABABBBBA..   | 116 | 101 | 49.75 | 2.13 |
| 61 <sup>st</sup> 200 | ABAAAAABABBAABABAB....      | 118 | 101 | 49.75 | 2.41 |
| 62 <sup>nd</sup> 200 | ABABAABBBBABBBBABA.....     | 117 | 101 | 49.75 | 2.27 |
| 63 <sup>rd</sup> 200 | AABABBAABABABBAABAA...      | 99  | 101 | 49.75 | 0.28 |
| 64 <sup>th</sup> 200 | AABAABABABBBBBAABABBB...    | 101 | 101 | 49.75 | 0    |
| 65 <sup>th</sup> 200 | AAAABABABABAABBAABBA... ..  | 119 | 101 | 49.75 | 2.55 |
| 66 <sup>th</sup> 200 | BAABBBBAAAABBAABBBB..       | 85  | 101 | 49.75 | 2.27 |
| 67 <sup>th</sup> 200 | AABAABABBBABBBABBBBA...     | 111 | 101 | 49.75 | 1.42 |
| 68 <sup>th</sup> 200 | AAAABABBBABAABAAABAAAA...   | 87  | 101 | 49.75 | 1.98 |
| 69 <sup>th</sup> 200 | BAABABAABAAAABABABBB...     | 90  | 101 | 49.75 | 1.56 |
| 70 <sup>th</sup> 200 | BAAABBABBBABBBABBBBAAB...   | 109 | 101 | 49.75 | 1.13 |
| 71 <sup>st</sup> 200 | AAABABBBAAABBAABBAABBB...   | 107 | 101 | 49.75 | 0.85 |
| 72 <sup>nd</sup> 200 | AABBBBBAABAABAAAABBA...     | 89  | 101 | 49.75 | 1.70 |

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| 74 <sup>th</sup> 200 | AABABBBABAABABBABBBBAAA... | 115 | 101 | 49.75 | 1.99 |
| 75 <sup>th</sup> 200 | ABAABBBBAAAABBAABABAB....  | 101 | 101 | 49.75 | 0    |

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