



INTER PARTICLE SPACING EFFECTS ON CONVECTIVE HEAT AND MASS TRANSFER THROUGH A NANO-FLUID IN HORIZONTAL CYLINDRICAL ANNULUS

Dr. G. V. P. N. Srikanth* & Dr. G. Srinivas**

* Associate Professor, Department of Mathematics, Guru Nanak Institute of Technology,
Hyderabad, Telangana

** Professor, Department of Mathematics, Guru Nanak Institute of Technology,
Hyderabad, Telangana

Abstract:

Particle size plays vital role in determination of heat and Mass transfer through a nano-fluid. But the chemical reaction of copper metal particle with water certainly alters the expected heat transfer rate. So the size of the nano-particle in its flow can be described by Newton's law. All the assumptions lead to a system of coupled non-linear governing partial differential equations. These equations are solved for velocity (w), temperature (θ) and Diffusion (C) using R-K 6th order with shooting method subject to the boundary conditions. The profiles of w , θ , C and the heat transfer rate (Nu) for various parameters are displayed. The inter particle spacing enhances the heat transfer rate due to the Brownian motion of the particles.

Index Terms: Cu-Water Nano-Fluid, MHD, Heat and Mass Transfer, Inter Particle Spacing, Chemical Reaction & R-K 6th order.

1.Introduction:

Some of the important applications of convective cooling are microelectronics, transportation, nuclear power plants, cooling of microchips in computers, and heat exchangers [2]. In all these the fluid carriers are cylindrical. Ellahi et al. [4] presented series solutions for magneto hydrodynamic flow of non-Newtonian nano fluid and heat transfer in coaxial porous cylinder with slip conditions. Also in another research, Ellahi et al. [5] presented series solutions of non-Newtonian nano fluids with Reynolds' model and Vogel's model by means of the homotopy analysis method. Non Newtonian nano fluid flow through a porous medium between two coaxial cylinders with heat transfer and variable viscosity was studied by Ellahi et al. [6]. The effects of MHD and temperature dependent viscosity on the flow of non-Newtonian nano fluid in a pipe were studied analytically by Ellahi et al. [7].

A numerical study on the flow and heat transfer across a confined square cylinder in the steady flow regime has been performed by Dhiman et al. [3]. Their results showed that the average Nusselt number increases monotonically with an increment in Reynolds number and/or Prandtl number. Furthermore, as Reynolds number increases, the length of the recirculation region increases. The Effect of thermal buoyancy on vortex shedding behind a square cylinder has been studied by Chatterjee and Mondal [1]. Also, Yoon et al. [16] investigated the flow past a square cylinder with different incidence angles. It was found that for steady flow, the wake length increases with increasing Reynolds number. Another numerical study on the forced convection heat transfer from a rectangular cylinder was performed by Rahnema et al. [12]. They found that increasing aspect ratio decreases Nusselt number for all Reynolds numbers. They also found that Nusselt number increases by increasing Reynolds number. Mixed convection inside a rectangular enclosure, filled with nano fluid, was studied by

Mahmoodi [11]. Soltanipour et al. [14] studied the heat transfer enhancement using $\gamma-Al_2O_3-H_2O$ nano fluid in a curved duct.

Another numerical study on fluid flow and heat transfer around a solid circular cylinder utilizing nano fluid was done by Valipour and Zare Ghadi [15]. Their results showed that as the solid volume fraction increases, the magnitude of minimum velocity in the wake region and recirculation length increases but separation angle decreases. A study on unconfined nano fluid flow and heat transfer around a square cylinder has been performed by Etminan-Farooji et al. recently [8]. They focused on the effects of Peclet number and types of nano fluids on heat transfer from the cylinder rather than fluid flow hydrodynamics. Sarkar et al. [13] studied vortex structure distributions and mixed convective heat transfer around a solid circular cylinder utilizing nano fluid for unsteady regime. Their result showed that the Strouhal number increases by increasing solid volume fraction. It also showed that increase in Strouhal numbers leads to reduction in vortex detachment. The core of the vortices grows during subsequent processes of vortex shedding cycle.

All the above could not describe the convective heat and mass transfer through a cylindrical annulus by considering the size of the nano particle and inter particle spacing. So in the present study the convective heat and mass transfer through cu-water nano fluid in a circular cylindrical annulus has considered. Graham [9], Jang-Choi [10] models are considered. Due to the presence of size of the particle and inter particle spacing in these models. The other physical phenomenon chemical reaction (K), magnetic parameter (M), gap of the annulus (s), heat source parameter (Q_H), porous parameter ($1/D$) are also taken into account.

2. Mathematical Formulation:

We consider convective flow of Cu-water nano-fluid through a circular cylindrical annulus filled with porous medium in the presence of heat source, with inner and outer walls are maintained at a constant temperature also the concentration is constant on the both walls. The flow is fully developed in the axial direction (z). The fluid region has constant physical properties and the flow is a convection flow taking place under thermal and molecular buoyancies and uniform axial pressure gradient. The Boussinesq approximation is invoked so that the density variation is confined to the thermal and molecular buoyancy forces. In the momentum, energy and diffusion are coupled and non-linear. Also the flow is unidirectional along the axial cylindrical annulus. The transverse magnetic field is applied the cylindrical annulus is assumed to be filled with porous material. The temperature dependent heat source is also considered. The geometry of the problem is shown in Fig.1.1.

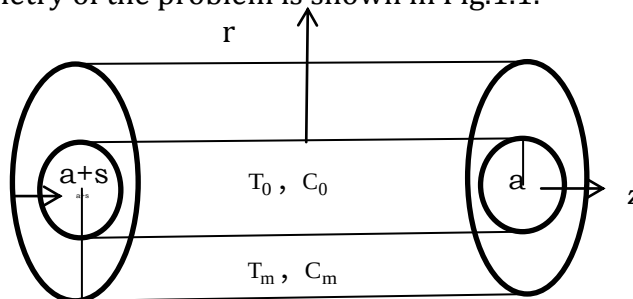


Figure 1.1: Schematic Diagram

Making use of the above assumptions the governing equations are

$$\frac{\partial}{\partial r} (r u) = 0 \quad (1)$$

$$\mu_{nf} \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) - \frac{\mu_{nf}}{k} w + \frac{\sigma B_0^2}{r^2} w + g(\rho\beta_T)_{nf} (T - T_0) + g(\rho\beta_c)_{nf} (c - c_0) = 0 \quad (2)$$

$$w \frac{\partial T}{\partial z} = \frac{\alpha_{nf}}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) - \frac{Q}{(\rho c_p)_{nf}} (T - T_0) \quad (3)$$

$$w \frac{\partial c}{\partial z} = \frac{D_{nf}}{r} \frac{\partial}{\partial r} \left(r \frac{\partial c}{\partial r} \right) + k_l (c - c_0) \quad (4)$$

The appropriate initial and boundary conditions for the problem are given by

$$T = T_0, \quad c = c_0, \quad w = 0 \quad \text{on} \quad r = a$$

$$T = T_m, \quad c = c_m, \quad w = 0 \quad \text{on} \quad r = a + s \quad (5)$$

Thermo-Physical properties are related as follows:

$$\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}$$

$$(\rho c_p)_{nf} = (1 - \phi) (\rho c_p)_f + \phi (\rho c_p)_s$$

$$(\rho\beta)_{nf} = (1 - \phi) (\rho\beta)_f + \phi (\rho\beta)_s$$

$$\frac{\mu_{nf}}{\mu_f} = 1 + 2.5 \phi + 4.5 \left[\frac{h}{d_p} \left(2 + \frac{h}{d_p} \right) \left(1 + \frac{h}{d_p} \right)^2 \right]^{-1}$$

$$k_{nf} = k_f (1 - \phi) + \beta_1 k_p \phi + c_1 \frac{d_f}{d_p} k_f \text{Re}_{d_p}^2 \text{Pr} \phi$$

Due to chemical reaction by the solvent and the metal particle the rate of change of the size of the Particle can be considered as:

$$\frac{dd_p}{dt} = -K d_p \Rightarrow d_p = d_0 e^{-K t} \quad \text{Where } d_p \text{ the diameter of the metal particle, } d_0 \text{ is the}$$

initial size of the particle (assumed as 50 nm) and K is the reaction rate:

$$\frac{\mu_{nf}}{\mu_f} = 1 + 2.5 \phi + 4.5 \left[\left(\frac{h e^{K t}}{50} \right) \left(2 + \frac{h e^{K t}}{50} \right) \left(1 + \frac{h e^{K t}}{50} \right)^2 \right]^{-1}$$

$$k_{nf} = k_f (1 - \phi) + \beta_1 k_p \phi + \frac{c_1}{50} d_f e^{K t} k_p \text{Re}_{d_p}^2 \text{Pr} \phi; \quad d_f = 0.384 \text{ nm for water} \quad (6)$$

where $\beta_1 = 0.01$ is a constant for considering the kapitza resistance per unit area

$$\text{Re}_{d_p} = \frac{d_p}{\nu_f} \frac{\kappa T}{3\pi \mu_f d_p l_f} = \frac{1.381 \times 10^{23} T}{\nu_f 3\pi \mu_f (0.738)}, \quad \text{Pr} = \text{Prandtl number} = \frac{\nu_f}{\alpha_f}; \quad T = 300 \text{ K}$$

$c_1 = 18 \times 10^6$ is proportionality constant

We introduce the following dimensionless variables:

$$R = \frac{r}{a}, \quad s^* = \frac{s}{a}, \quad Z = \frac{z}{a}, \quad W = \frac{w}{v_f a}, \quad U = \frac{u}{v_f}, \quad \theta = \frac{T - T_0}{T_m - T_0}, \quad C = \frac{c - c_0}{c_m - c_0} \quad (7)$$

Using equations (5),(6),(7), the Equations(2),(3)&(4) can be written in the following dimensionless form:

$$\left(1 + 2.5\phi + 4.5 \left[\left(\frac{he Kt}{50} \right) \left(2 + \frac{he Kt}{50} \right) \left(1 + \frac{he Kt}{50} \right)^2 \right]^{-1} \right) \left(\frac{\partial^2 W}{\partial R^2} + \frac{1}{R} \frac{\partial W}{\partial R} \right) \left(1 - \phi + \phi \left(\frac{\rho_s}{\rho_f} \right)^{-1} \right) - \frac{1}{D} \left(1 + 2.5\phi + 4.5 \left[\left(\frac{he Kt}{50} \right) \left(2 + \frac{he Kt}{50} \right) \left(1 + \frac{he Kt}{50} \right)^2 \right]^{-1} \right) \left(1 - \phi + \phi \left(\frac{\rho_s}{\rho_f} \right)^{-1} \right) W$$

$$- \left(1 - \phi + \phi \left(\frac{\rho_s}{\rho_f} \right)^{-1} \right) MW + \left(1 - \phi + \phi \left(\frac{\rho_s}{\rho_f} \right)^{-1} \right) Gr \theta + \left(1 - \phi + \phi \left(\frac{\rho_s}{\rho_f} \right)^{-1} \right) Gc C = 0$$

$$W \frac{\partial \theta}{\partial z} = \frac{1}{Pr} \left[\left(1 - \phi + 0.01\phi \frac{k_p}{k_f} + \frac{k_p}{k_f} \frac{\rho_f^2 c_{pf} e^{Kt}}{50 \mu_f^3} - 28632.9991 \times 10^{-52} \right) \left(1 - \phi + \phi \left(\frac{\rho c_p}{\rho c_p} \right)^{-1} \right) \left[\frac{\partial^2 \theta}{\partial R^2} + \frac{1}{R} \frac{\partial \theta}{\partial R} \right] - \frac{1}{Pr} Q_H \left[\left(1 - \phi + \phi \left(\frac{\rho c_p}{\rho c_p} \right)^{-1} \right) \theta \right] \right]$$

$$W \frac{\partial C}{\partial z} = \frac{1}{Sc} \left[\frac{\partial^2 C}{\partial R^2} + \frac{1}{R} \frac{\partial C}{\partial R} \right] + KC$$

Where the corresponding boundary conditions (5) can be written in the dimensionless form as:

$$W = 0, \theta = 0, C = 0 \quad \text{on } R = 1$$

$$W = 0, \theta = 1, C = 1 \quad \text{on } R = 1+s^* \quad (\text{We drop } * \text{ in the later part})$$

Here Pr is the Prandtl number, M is the magnetic parameter (Hartmann number), Q_H is the heat source parameter, Sc is the Schmidt number, K is the chemical Reaction parameter, D^{-1} is the Darcy number, Gr is the thermal Grashof number, Gc is the Molecular Grashof number, which are defined as:

$$Pr = \frac{\nu_f}{\alpha_f}, \quad M = \frac{\sigma B_0^2}{\mu_f} a, \quad Q_H = \frac{Q}{k_f} a, \quad Sc = \frac{\nu_f}{D_{nf}}, \quad K = \frac{k_l}{\nu_f} a, \quad \frac{1}{D} = \frac{a}{k}$$

$$Gr = \frac{g \beta_f a^3 (T_m - T_0)}{\nu_f^2}, \quad Gc = \frac{g \beta_f a^3 (c_m - c_0)}{\nu_f^2}$$

The local Nusselt number Nu in dimension less form:

$$Nu = - \frac{k_{nf}}{k_f} \theta' (1+s)$$

The local Sherwood number Sh in dimension less form:

$$Sh = - \frac{\nu_f}{D_{nf}} C' (1+s)$$

3. Solution of the Problem:

The typical cross section of the cylinder has considered for numerical computations. The governing equations are solved for velocity (w), temperature (θ) and diffusion (C) by using R-K 6th order with Shooting methods with the help of Mathematica package across the cylindrical annulus subject to the boundary conditions. During the solution the axial temperature and constriction gradients are assumed to be constant. The standard values throughout the computation are: $\phi = 0.02$, $S = 0.5$, $h = 4$, $Gc = 5$, $Gr = 5$, $M = 5$, $Q_H = 5$, $Sc = 0.6$, $K = 0.5$, $1/D = 5$.

4. Discussion of Results:

The effect of various parameters viz. solid volume fraction (ϕ), inter particle spacing (h), magnetic parameter (M), gap of the annulus (s), heat source parameter (Q_H), porous parameter (D), thermal Grashof number (Gr), Molecular Grashof number (Gc), Schmidt number (Sc) and chemical reaction parameter (K) on velocity (W), temperature (θ) and diffusion (C) in the cylindrical annulus are exhibited in graphs from Figures 1 to

23. The other parameters were assumed constant. The Prandtl Number (Pr) kept constant as 7 (for water).

From Fig. 1-10 the profiles of velocity are parabolic in shape indicating that flow is maximum in the mid region of the annulus for all parameters. The enhancement of solid volume fraction (ϕ) enhances the flow due to the Brownian motion (Fig. 1). The absence of the nano-particles leads to the almost constant flow of the fluid. The variation of the ϕ up to 5% has been studied and found that flow varies rapidly. The velocity profile for gap of the annulus (s) is depicted in Fig.2. As the gap increases, the velocity (W) increases significantly. The flow is significantly high when the radius of the outer cylinder is double the radius of the inner cylinder. The natural flow of the fluid dominating the adhesive property of the liquid like layer around the particle. This is leading to the increase in the velocity of the fluid with increase in the thickness of the layer in the closed geometry (Fig.3). The significant enhancement of velocity (W) is also observed for inter particle spacing (h) from 2nm to 10nm. The increase in molecular (G_c) or thermal (G_r) buoyancy increases the velocity (Fig. 4&5). Also observed that the rate of increase in G_r and G_c uniformly increases the velocity. From Fig. 6 the increase in Lorentz force retards the flow; the variation of velocity is very less due to the presence of the metal particles. The variation of velocity with heat source (Q_H) is exhibited in Fig. 7. The increase in temperature dependent heat source retards the flow slightly. This slight variation is due to the presence of the metal particles, which are affected due to the heat source very much. Fig. 8 exhibits the variation of velocity with the Schmidt number (Sc). Velocity slightly increases with increase in Sc . Fig. 9 exhibits the variation of the velocity with chemical reaction parameter (K). Velocity is more when the reaction is destructive and it less when the reaction is generative. The velocity is maximum in mid region of annulus and has quick convergence when we move slightly from the mid region to the boundary. The velocity profile for Darcy parameter (D^{-1}) is in Fig. 10. The velocity decreases gradually with increase in D^{-1} .

The temperature profiles are depicted from Fig. 11 to 20. All these profiles are linear for all variations, indicating that the temperature gradually decreases from outer cylinder to inner cylinder in the annulus. From Fig.11 the increase in solid volume fraction (ϕ) enhances the temperature. From Fig.12 the rapid reduction of temperature is observed with the reduction of the gap between the inner and outer cylinders (s). The variation of temperature is found absolutely uniform throughout the annulus when the radius of the outer cylinder is double the radius of inner cylinder. Fig. 13 exhibits the variation of temperature with inter particle spacing (h). The increase in h (2nm to 10nm) decreases the temperature due to spacing in between the particles. From Fig. 14 & 15, molecular (G_c) and thermal (G_r) buoyancy reduces the temperature gradually. The magnetic parameter (M) increases the temperature slightly (Fig. 16). From Fig. 17 the increase in heat source (Q_H) reduces the temperature slightly due to the presence of nano-particles, absorbing the heat. Fig. 18 exhibits that the temperature is directly proportional to the diffusivity. The variation of temperature is low for variation of Schmidt number (Sc). Fig. 19 shows the temperature profile for variation of chemical reaction parameter (K). Temperature falls from generative to destructive reaction gradually. From Fig. 20 temperature is inversely proportional to the porosity of the medium. The variation of temperature is low for variation of D^{-1} for 2 to 10.

The diffusion profiles are depicted from Fig. 21 to 23. Fig. 21 exhibits the variation of diffusion is low for variation of Schmidt number (Sc). Fig. 22 shows the diffusion profile for variation of chemical reaction parameter (K). Diffusion rises from generative to destructive reaction gradually. From Fig. 23 diffusion is inversely

proportional to the porosity of the medium. The variation of diffusion is low for variation of D^{-1} for 2 to 10.

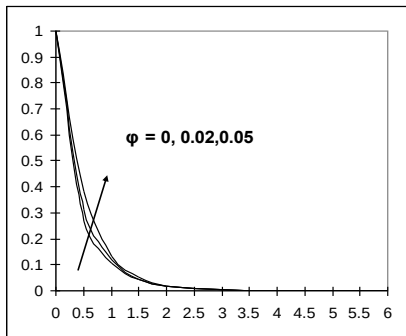


Fig.1 Variation of U with ϕ

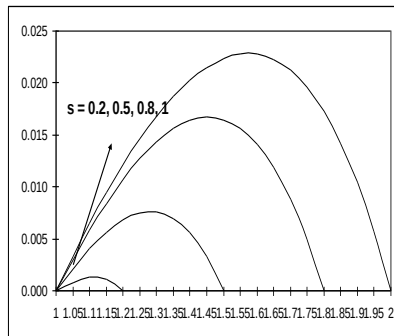


Fig.2 Variation of w with s

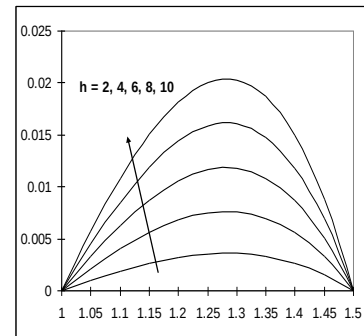


Fig.3 Variation of w with h

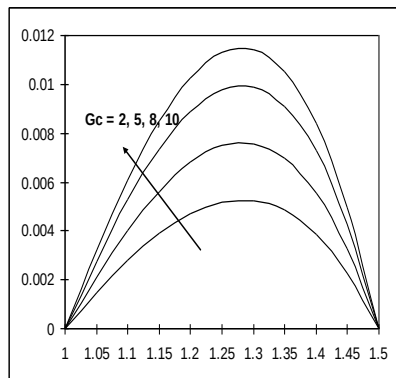


Fig.4 Variation of w with G_c

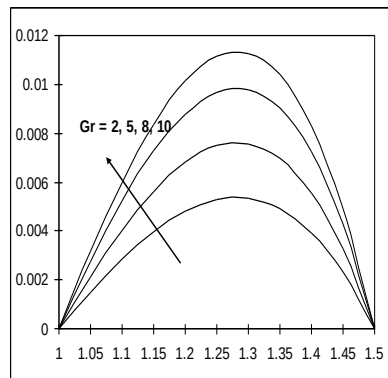


Fig.5 Variation of w with Gr

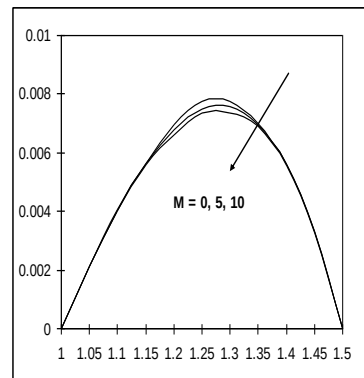


Fig.6 Variation of w with M

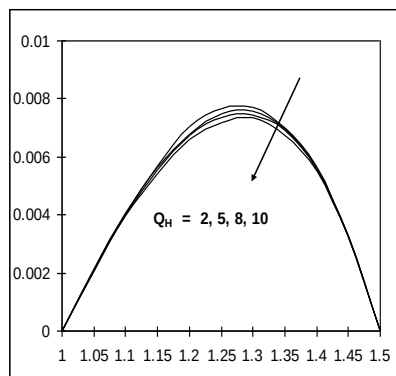


Fig.7 Variation of w with Q_H

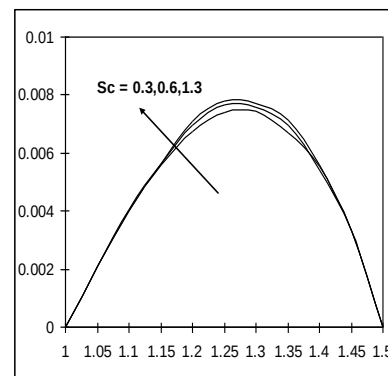


Fig.8 Variation of w with Sc

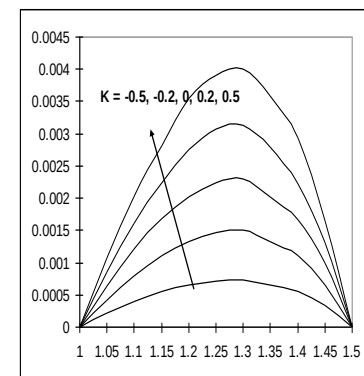


Fig.9 Variation of w with K

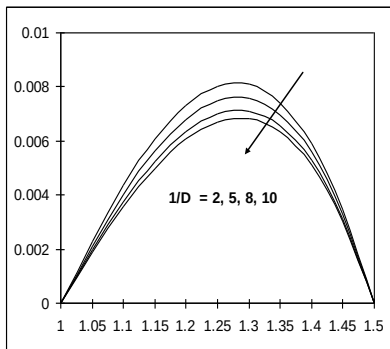


Fig.10 Variation of w with D^{-1}

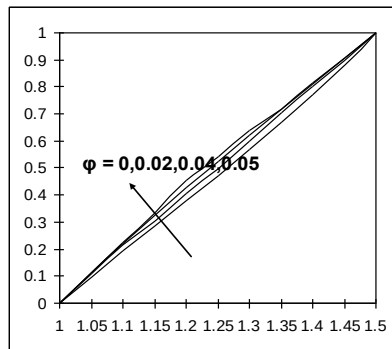


Fig.11 Variation of θ with ϕ

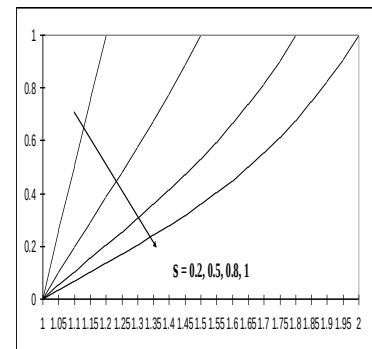


Fig.12 Variation of θ with s

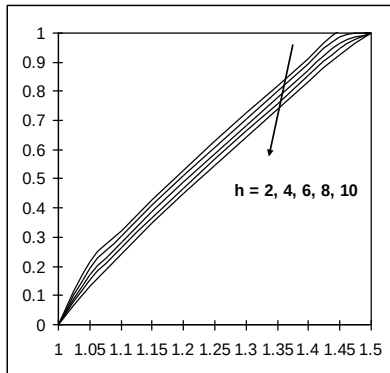


Fig.13 Variation of θ with h

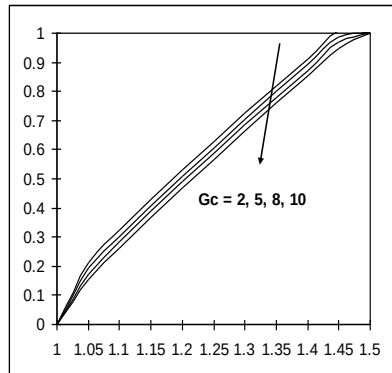


Fig.14 Variation of θ with G_c

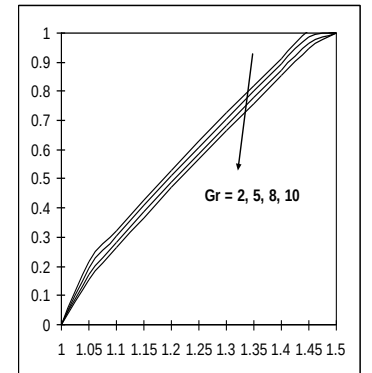


Fig.15 Variation of θ with Gr

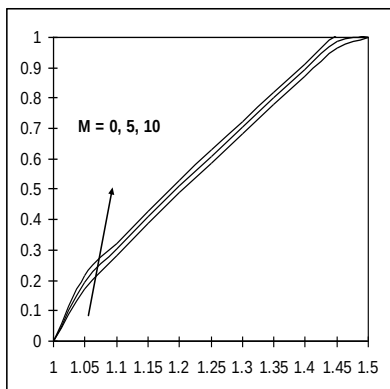


Fig.16 Variation of θ with M

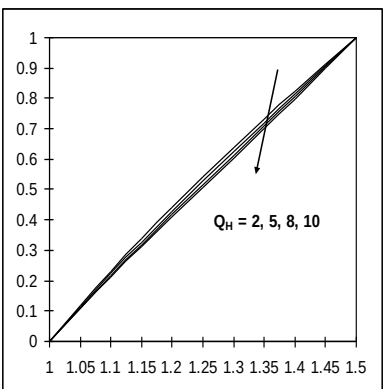


Fig.17 Variation of θ with Q_H

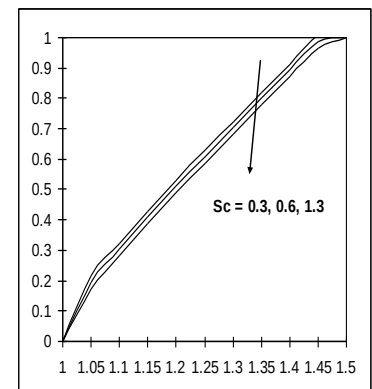


Fig.18 Variation of θ with Sc

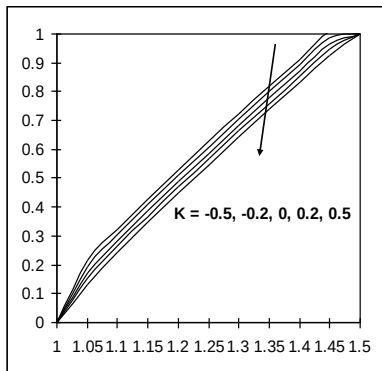


Fig.19 Variation of θ with K

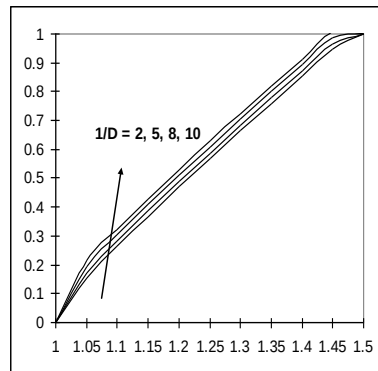


Fig.20 Variation of θ with D^{-1}

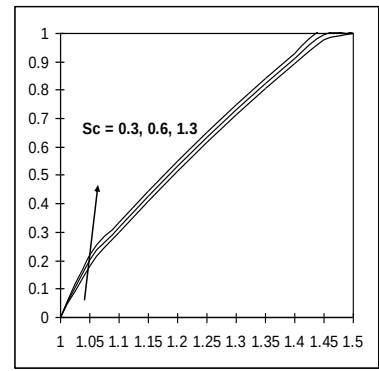


Fig.21 Variation of C with Sc

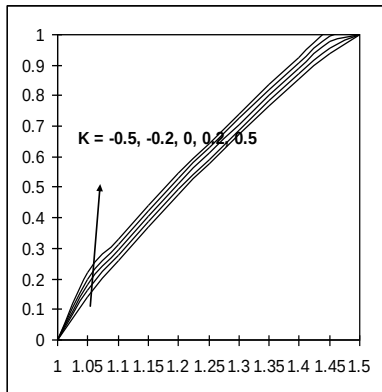


Fig.22 Variation of C with K

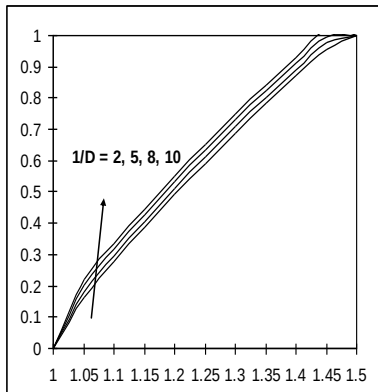


Fig.23 Variation of C with D^{-1}

ϕ	$h = 2,$ $Q_H = 5$	$h = 5,$ $Q_H = 5$	$h = 10,$ $Q_H = 5$
0	2.42074	2.42406	2.43088
0.05	2.91288	2.91627	2.92313
0.1	3.40956	3.41299	3.41981

Table-1: Nusselt Number

Table-1 display the heat transfer (Nusselt number) for ϕ , h , Q_H calculated on the outer cylinder. The entire table signifies the importance of nano fluid from the second and third rows ($\phi = 5\%$ and 10% respectively). When compared with first row (non-nano fluid). Interestingly the increase in the spacing among the metal particles slightly enhances the heat transfer this may be due to the porous medium objecting the interaction of the particles with outer cylinder. Naturally the heat source enhances the heat transfer.

References:

1. Chatterjee, D., Mondal, B., Effect of thermal buoyancy on vortex shedding behind a square cylinder in cross flow at low Reynolds numbers, International Journal of Heat & Mass Transfer, 54 (2011), 21-22, pp. 5262–5274.

2. Das, S. K., Choi, S. U. S., Yu, W. H., Pradeep, T., Nano fluids: Science and Technology, John Wiley & Sons, Hoboken, NJ, USA, 2008.
3. Dhiman, A. K., Chhabra, R. P., Eswaran, V., Flow and Heat Transfer Across a Confined Square Cylinder in the Steady Flow Regime: Effect of Peclet Number, International Journal of Heat & Mass Transfer, 48 (2005), 21-22, pp. 4598-4614.
4. Ellahi, R., Zeeshan, A., Vafai, K., Rahman, H., Series solutions for magnetohydrodynamic flow of non-Newtonian nano fluid and heat transfer in coaxial porous cylinder with slip conditions, J Nano engineering and Nano systems, 225 (2011), 3, pp. 123-132.
5. Ellahi, R., Raza, M., Vafai, K., Series solutions of non-Newtonian nano fluids with Reynolds' model and Vogel's model by means of the homotopy analysis method, Mathematical and Computer Modeling, 55 (2012), 7-8, pp. 1876-1891
6. Ellahi, R., Aziz, S., Zeeshan, A., Non Newtonian nano fluid flow through a porous medium between two coaxial cylinders with heat transfer and variable viscosity, Journal of Porous Media, 16 (2013), 3, pp. 205-216
7. Ellahi, R., The effects of MHD and temperature dependent viscosity on the flow of non-Newtonian nano fluid in a pipe: Analytical solutions, Applied Mathematical Modeling, 37 (2013), 3, pp. 1451-1467
8. Etminan-Farooji, V., Ebrahimnia-Bajestan, E., Niazmand, H., Wongwises, S., Unconfined laminar nano fluid flow and heat transfer around a square cylinder, International Journal of Heat and Mass Transfer, 55 (2012), 5-6, pp. 1475-1485
9. Graham A.L., On the viscosity of suspension of solid spheres, Appl. Sci. Res., 37 (3), 275-286, (1981).
10. Jang S.P. and Choi S.U.S., Effects of various parameters on nano-fluid thermal conductivity, J. Heat Transf., 129(5), 617– 623, (2007).
11. Mahmoodi, M., Mixed convection inside nano fluid filled rectangular enclosures with moving bottom wall, Thermal science, 15 (2011), 3, pp. 889-903
12. Rahnema, M., Hashemian, S. M., Farhadi, M., Forced convection heat transfer from a rectangular cylinder: effect of aspect ratio, 16th international symposium on transport phenomena, Prague, Chec, 2005, pp. 1-5.
13. Sarkar, S., Ganguly, S., Biswas, G., Mixed convective heat transfer of nano fluids past a circular cylinder in cross flow in unsteady regime, International Journal of Heat and Mass Transfer, 55 (2012), 17-18, pp.4783-4799.
14. Soltanipour, H., Choupani, P., Mirzaee, I., Numerical analysis of heat transfer enhancement with using γ -Al₂O₃-H₂O nano fluid and longitudinal ribs in a curved duct, Thermal science, 16 (2012), 2, pp. 469-480.
15. Valipour, M. S., Zare Ghadi, A., Numerical investigation of fluid flow and heat transfer around a solid circular cylinder utilizing nano fluid, International Communications in Heat and Mass Transfer, 38 (2011), 9, pp. 1296-1304.
16. Yoon, D., Yangl, K., Choi, C., Flow past a square cylinder with an angle of incidence, Physics of fluids, 22 (2010).