



FUZZY SOFT GAMMA REGULAR SEMIGROUPS

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Abstract:

In this paper, we have discussed about the fuzzy soft Γ -regular and Γ -intra regular semigroups and their properties.

Index Terms: Soft semigroups, soft Γ -ideals, Γ -regular semigroups, Γ -intra regular semigroups, soft Γ -regular semigroups & fuzzy soft Γ -intra regular semigroups.

1. Introduction:

The concept of fuzzy set was introduced by Zadeh [15] in 1965. Sen and Saha [13] defined the Γ -semigroup in 1986. Soft set theory proposed by Molotsov [8] in 1999. Soft set theory has been applied in many fields. Maji et al [7] worked on soft set theory and fuzzy soft set theory. Ali et al [1] introduced new operations on soft sets. Chinram and Jirojkul [4] defined the bi-ideals in Γ -semigroups in 2007. Prince et al [12] presented the fuzzy Γ -bi-ideals in Γ -semigroups in 2009. Kehayopula et al [6] initiated the bi-ideals and quasi ideals of semigroups and ordered semigroups. Dheena et al [5] studied the characterization of regular Γ -semigroups through fuzzy ideals in 2007. Chinnadurai et al [3] worked on interval valued fuzzy soft semigroups, and he studied Γ -regular semigroups in fuzzy ideals in algebraic structures. Muhammad Ifran Ali [10] studied soft ideals over semigroups. Sujit Kumar Sardar [14] discussed the properties of fuzzy ideals in Γ -semigroups. Muhammad Akaram et al [9] discussed Fuzzy soft Γ semigroups. Munazza Naz et al [11] worked fuzzy soft quasi-ideals on fuzzy soft semigroups Ali et al [2] studied soft ideals and generalized fuzzy ideals in semigroups in 2009.

2. Preliminaries:

Definition 2.1 [13]: Let $S = \{a, b, c, \dots\}$ and $\Gamma = \{\alpha, \beta, \gamma, \dots\}$ be two non-empty sets. Then S is called a Γ -semigroup if it satisfies the conditions

$$(i) \quad a\alpha b \in S,$$

$$(ii) \quad (a\beta b)\gamma c = a\beta(b\gamma c) \quad \forall a, b, c \in S \text{ and } \alpha, \beta, \gamma \in \Gamma.$$

Definition 2.2 [5]: A Γ -semigroup S is called a regular if for each element $a \in S$, there exists $x \in S$ and $\alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a$.

Definition 2.3 [8]: Let U be the universal set, E be the set of parameters, $P(U)$ denote the power set of U and A be a non-empty subset of E . A pair (F, A) is called a soft set over U , where F is mapping given by $F : A \rightarrow P(U)$.

Definition 2.4 [7]: Let (F, A) and (G, B) be two soft sets over a common universe U then (F, A) AND (G, B) denoted by $(F, A) \wedge (G, B)$ is defined as

$$(F, A) \wedge (G, B) = (H, A \times B) \text{ where } H(\alpha, \beta) = F(\alpha) \cap G(\beta), \quad \forall (\alpha, \beta) \in A \times B$$

Definition 2.5 [7]: Let (F, A) and (G, B) be two soft sets over a common universe U then (F, A) OR (G, B) denoted by $(F, A) \vee (G, B)$ is defined as $(F, A) \vee (G, B) = (H, A \times B)$ where $H(\alpha, \beta) = F(\alpha) \cup G(\beta) \quad \forall (\alpha, \beta) \in A \times B$.

Definition 2.6 [1]: The extended union of two fuzzy soft sets (F, A) and (G, B) over a common universe U is fuzzy soft set denoted by $(F, A) \cup_{\epsilon} (G, B)$ defined as

$$(F, A) \cup_{\epsilon} (G, B) = (H, C) \text{ where } C = A \cup B, \quad \forall c \in C.$$

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - B \\ G(c) & \text{if } c \in B - A \\ F(c) \cup G(c) & \text{if } c \in A \cap B. \end{cases}$$

Definition 2.7 [1]: The extended intersection of two fuzzy soft sets (F, A) and (G, B) over a common universe U is fuzzy soft set denoted by $(F, A) \cap_{\epsilon} (G, B)$ defined as $(F, A) \cap_{\epsilon} (G, B) = (H, C)$ where $C = A \cup B$, $\forall c \in C$.

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - B \\ G(c) & \text{if } c \in B - A \\ F(c) \cap G(c) & \text{if } c \in A \cap B. \end{cases}$$

Definition 2.8 [1]: Let (F, A) and (G, B) be two soft sets over a common universe U such that $A \cap B \neq \phi$. The restricted intersection of (F, A) and (G, B) is denoted by $(F, A) \cap_R (G, B)$ and defined as $(F, A) \cap_R (G, B) = (H, C)$ where $C = A \cap B$, $\forall c \in C$, where $H(c) = F(c) \cap G(c)$.

Definition 2.9 [2]: The restricted product (H, C) of two fuzzy soft sets (F, A) and (G, B) over a semigroup S is defined as $(H, C) = (F, A) \tilde{\sim} (G, B)$ where $C = A \cap B$ by $H(c) = F(c) \tilde{\sim} G(c)$, $\forall c \in C$.

Definition 2.10 [3]: A soft set (F, A) is called a soft semigroup over S if $(F, A) \tilde{\sim} (F, A) \subseteq (F, A)$. Clearly a soft set (F, A) over a semigroup S is a soft semigroup if and only if $\phi \neq F(a)$ is a subsemigroup of S , $\forall a \in A$.

Definition 2.11 [10]: A soft semigroup (F, A) over a semigroup S is called a soft regular semigroup if for each $\alpha \in A$, $F(\alpha)$ is regular.

Definition 2.12 [15]: Let X be non-empty set. A fuzzy subset μ of X is a function from X into the closed unit interval $[0, 1]$. The set of all fuzzy subsets of X is called a fuzzy power set of X and is denoted by $FP(X)$.

Definition 2.13 [9]: A fuzzy soft set (μ, A) of a Γ -semigroup S , then (μ, A) is called a fuzzy soft Γ -subsemigroup of S if $\mu_a(x\gamma y) \geq \min\{\mu_a(x), \mu_a(y)\}$, $\forall a \in A, x, y \in S, \gamma \in \Gamma$.

Definition 2.14 [9]: A fuzzy soft set (μ, A) of a Γ -semigroup S is called a fuzzy soft Γ -left(right) ideal of S if $\mu_a(x\gamma y) \geq \mu_a(y)$, $(\mu_a(x\gamma y) \geq \mu_a(x))$, $\forall a \in A, x, y \in S$ and $\gamma \in \Gamma$.

Definition 2.15 [9]: A fuzzy soft Γ -subsemigroup (μ, A) of a Γ -semigroup S is called a fuzzy soft Γ -ideal of S if $\mu_a(x\alpha z\beta y) \geq \max\{\mu_a(x), \mu_a(y)\}$, $\forall a \in A, x, y, z \in S, \alpha, \beta \in \Gamma$.

Definition 2.16 [9]: A fuzzy soft Γ -subsemigroup (μ, A) of a Γ -semigroup S is called a fuzzy soft Γ -bi-ideal of S if $\mu_a(x\alpha z\beta y) \geq \min\{\mu_a(x), \mu_a(y)\}$, $\forall a \in A, x, y, z \in S, \alpha, \beta \in \Gamma$.

Definition 2.17 [9]: A fuzzy soft Γ -subsemigroup (μ, A) of a Γ -semigroup S is called a fuzzy soft Γ -interior ideal of S if $\mu_a(x\alpha z\beta y) \geq \mu_a(z)$, $\forall a \in A, x, y, z \in S$ and $\alpha, \beta \in \Gamma$.

Definition 2.18 [11]: A fuzzy soft (μ, A) over S is said to be a fuzzy soft quasi ideal of S if $\mu(e)$ is a fuzzy quasi ideal of S $\forall e \in A$.

Definition 2.19 [11]: A fuzzy soft (μ, A) over S is said to be a fuzzy soft generalized bi-ideal of S if $\mu(e)$ is a fuzzy generalized bi-ideal of S $\forall e \in A$.

3. Fuzzy Soft Gamma Regular Semigroups:

In This section S denotes the soft Γ -regular semigroup.

Definition 3.1: A soft Γ -semigroup (F, A) over a semigroup S is called a soft Γ -regular semigroup if for each $\alpha, \beta \in A$, $F(\alpha, \beta)$ is regular.

Example 3.2: $S = \{a_1, a_2, a_3\}$ and $\Gamma = \{\alpha, \beta\}$ where α, β is defined on S with the following cayley table:

α	a_1	a_2	a_3	a_4
a_1	a_1	a_1	a_1	a_1
a_2	a_1	a_2	a_3	a_4
a_3	a_1	a_3	a_3	a_3
a_4	a_1	a_3	a_3	a_3

β	a_1	a_2	a_3	a_4
a_1	a_1	a_1	a_1	a_1
a_2	a_1	a_2	a_3	a_4
a_3	a_1	a_3	a_3	a_3
a_4	a_1	a_2	a_3	a_4

Table-1

Consider $E = \{a_1, a_2, a_3, a_4\}$ and $F(a_1) = \{a_1, a_3\}$, $F(a_2) = \{a_2, a_3\}$, $F(a_3) = \{a_1, a_2, a_3\}$, $F(a_4) = \{a_2, a_3, a_4\}$. Hence (F, S) is soft Γ -regular semigroup.

Theorem 3.3: Let (F, A) and (G, B) be two fuzzy soft Γ -ideal (bi-ideal, interior ideal) over soft Γ -regular semigroup S , then $(F, A) \wedge (G, B)$ is fuzzy soft Γ -ideal (bi-ideal, interior ideal) over soft Γ -regular semigroup S .

Proof: Let (F, A) and (G, B) be two fuzzy soft Γ -ideal over soft Γ -regular semigroup S . Now we defined $(F, A) \wedge (G, B) = (H, C)$ where $C = A \times B$ and $H(a, b) = F(a) \cap G(b) \quad \forall (a, b) \in C$.

Consider

$$\begin{aligned}
 \mu_{H(a,b)}(x\alpha y) &= (\mu_{F(a)} \cap \mu_{G(b)})(x\alpha y) \\
 &= \min\{\mu_{F(a)}(x\alpha y), \mu_{G(b)}(x\alpha y)\} \\
 &\geq \min\{\max\{\mu_{F(a)}(x), \mu_{F(a)}(y)\}, \max\{\mu_{G(b)}(x), \mu_{G(b)}(y)\}\} \\
 &= \max\{\min\{\mu_{F(a)}(x), \mu_{G(b)}(x)\}, \min\{\mu_{F(a)}(y), \mu_{G(b)}(y)\}\} \\
 &= \max\{(\mu_{F(a)} \cap \mu_{G(b)})(x), (\mu_{F(a)} \cap \mu_{G(b)})(y)\} \\
 &= \max\{\mu_{H(a,b)}(x), \mu_{H(a,b)}(y)\}
 \end{aligned}$$

Hence $(F, A) \wedge (G, B)$ is fuzzy soft Γ -ideal over soft Γ -regular semigroup S .

Theorem 3.4: Let (F, A) and (G, B) be two fuzzy soft Γ -ideal (bi-ideal, interior ideal) over soft Γ -regular semigroup S , then $(F, A) \vee (G, B)$ is fuzzy soft Γ -ideal (bi-ideal, interior ideal) over soft Γ -regular semigroup S .

Proof: The proof is straightforward.

Theorem 3.5: Let (F, A) and (G, B) be two fuzzy soft Γ -ideal of S , then $(F, A) \cap_{\subseteq} (G, B)$ is a fuzzy soft Γ -ideal over S .

Proof: Let (F, A) and (G, B) be two fuzzy soft Γ -ideal over S , then

$$\begin{aligned}
 (F, A) \cap_{\subseteq} (G, B) &= (H, C) \text{ where } C = A \cup B, \quad \forall c \in C \\
 H(c) &= \begin{cases} F(c) & \text{if } c \in A - B \\ G(c) & \text{if } c \in B - A \\ F(c) \cap G(c) & \text{if } c \in A \cap B. \end{cases}
 \end{aligned}$$

Let $s, t \in S$ and $\alpha \in \Gamma$.

(i) If $c \in A - B$

$$\begin{aligned}\mu_{H(c)}(s\alpha t) &= \mu_{F(c)}(s\alpha t) \\ &\geq \max\{\mu_{F(c)}(s), \mu_{F(c)}(t)\} \\ &= \max\{\mu_{H(c)}(s), \mu_{H(c)}(t)\}\end{aligned}$$

(ii) If $c \in B - A$

$$\begin{aligned}\mu_{H(c)}(s\alpha t) &= \mu_{G(c)}(s\alpha t) \\ &\geq \max\{\mu_{G(c)}(s), \mu_{G(c)}(t)\} \\ &= \max\{\mu_{H(c)}(s), \mu_{H(c)}(t)\}\end{aligned}$$

(iii) If $c \in A \cap B$ then $H(c) = \min\{F(c), G(c)\} = \{F(c) \cap G(c)\}$

Now verify that $H(c)(s\alpha t) \geq \max\{H(c)(s), H(c)(t)\}$, $\forall s, t \in S, \alpha \in \Gamma$ and $c \in C$. Thus

$\mu_{H(c)}(s\alpha t) \geq \max\{\mu_{H(c)}(s), \mu_{H(c)}(t)\}$. Hence $(F, A) \cap_{\in} (G, B) = (H, C)$ is a fuzzy soft Γ -ideal over S .

Theorem 3.6: Let (F, A) and (G, B) be two fuzzy soft Γ -ideal over S , $(F, A) \cup_{\in} (G, B)$ is a fuzzy soft Γ -ideal over S .

Proof: The proof is straightforward.

Theorem 3.7: Let (F, A) and (G, B) be two fuzzy soft sets of soft Γ -regular semigroup S . A_1 and A_2 are two non-empty subsets of S .

$$(i) \chi_{F(e)A_1} \tilde{\cap} \chi_{F(e)A_2} = \chi_{F(e)(A_1 \cap A_2)}$$

$$(ii) \chi_{F(e)A_1} \tilde{\circ} \chi_{F(e)A_2} = \chi_{F(e)(A_1 \Gamma A_2)}$$

Proof: Let $p \in S$, if $p \in A_1 \cap A_2$, then $p \in A_1$ and $p \in A_2$. we have

$$\begin{aligned}(\chi_{F(e)A_1} \tilde{\cap} \chi_{F(e)A_2})(p) &= \min\{\chi_{F(e)A_1}(p), \chi_{F(e)A_2}(p)\} \\ &= \min\{0, 0\} \\ &= 0 \\ &= \chi_{F(e)(A_1 \Gamma A_2)}(p)\end{aligned}$$

Suppose $p \notin A_1 \cap A_2$, then $p \notin A_1$ and $p \notin A_2$

$$\begin{aligned}(\chi_{F(e)A_1} \tilde{\cap} \chi_{F(e)A_2})(p) &= \min\{\chi_{F(e)A_1}(p), \chi_{F(e)A_2}(p)\} \\ &= \min\{0, 0\} \\ &= 0 \\ &= \chi_{F(e)(A_1 \Gamma A_2)}(p)\end{aligned}$$

Let $p \in S$, if $p \in A_1 \Gamma A_2$, then there exists $a_1 \in A_1, \gamma \in \Gamma$ and $a_2 \in A_2$, such that $p = a_1 \gamma a_2$

$$\begin{aligned}(\chi_{F(e)A_1} \tilde{\circ} \chi_{F(e)A_2})(p) &= \sup_{p=c\gamma d} \min\{\chi_{F(e)A_1}(c), \chi_{F(e)A_2}(d)\} \\ &\geq \min\{\chi_{F(e)A_1}(p), \chi_{F(e)A_2}(p)\} \\ &= \min\{1, 1\} \\ &= 1\end{aligned}$$

So $(\chi_{F(e)A_1} \tilde{\circ} \chi_{F(e)A_2})(p) = 1$ since $p \in A_1 \Gamma A_2$, $\chi_{F(e)(A_1 \Gamma A_2)}(p) = 1$. Suppose $p \notin A_1 \Gamma A_2$, then $p \notin a_1 \gamma a_2, a_1 \in A_1, \gamma \in \Gamma$ and $a_2 \in A_2$.

$$\begin{aligned}(\chi_{F(e)A_1} \tilde{\cap} \chi_{F(e)A_2})(p) &= \min\{\chi_{F(e)A_1}(p), \chi_{F(e)A_2}(p)\} \\&= \min\{0, 0\} \\&= 0 \\&= \chi_{F(e)(A_1 \Gamma A_2)}(p)\end{aligned}$$

Hence $\chi_{F(e)A_1} \tilde{\cap} \chi_{F(e)A_2} = \chi_{F(e)(A_1 \Gamma A_2)}$

Theorem 3.8: Let (F, A) be a soft subset of a semigroup S , (F, A) be a soft Γ – subsemigroup of S if and only if $\chi_{F(e)}$ is fuzzy soft Γ – subsemigroup of S .

Proof: Let (F, A) be a soft Γ – semigroup of S

$$\chi_{F(e)}\lambda(e) = \begin{cases} 1 & \text{if } a \in F(e) \\ 0 & \text{if } a \notin F(e). \end{cases}$$

Let $a, b \in S$, $\gamma \in \Gamma$ if $\chi_{F(e)}\lambda(a\gamma b) \leq \min\{\chi_{F(e)}\lambda(a), \chi_{F(e)}\lambda(b)\}$, then $\chi_{F(e)}\lambda(a) = 1$,

$\chi_{F(e)}\lambda(b) = 1$ and $\chi_{F(e)}\lambda(a\gamma b) = 0$, this implies that $a, b \in F(e)$ since $F(e)$ is Γ – subsemigroup of S , $a\gamma b \in F(e)$ and hence $\chi_{F(e)}\lambda(a\gamma b) = 1$ which is a contradiction. Thus $\chi_{F(e)}\lambda(a\gamma b) \leq \min\{\chi_{F(e)}\lambda(a), \chi_{F(e)}\lambda(b)\}$, $\forall a, b \in F(e)$ and $e \in A$. Conversely assume that $\chi_{F(e)}$ is soft Γ – subsemigroup of S . Let $a, b \in F(e)$, then $\chi_{F(e)}\lambda(a) = 1$, $\chi_{F(e)}\lambda(b) = 1$, $\chi_{F(e)}$ fuzzy soft Γ – subsemigroup. Now $\min\{\chi_{F(e)}\lambda(a), \chi_{F(e)}\lambda(b)\} = 1 \leq \chi_{F(e)}\lambda(a\gamma b)$, this implies that $\chi_{F(e)}\lambda(a\gamma b) = 1$ and hence $a, b \in F(e) \forall e \in A$. Therefore (F, A) is a soft Γ – subsemigroup of S .

Theorem 3.9: Let (F, B) be a soft Γ – bi-ideal of S if and only if $\chi_{F(e)B}$ (characteristic function) is a fuzzy soft Γ – bi-ideal of S .

Proof: Assume that (F, B) be a soft Γ – bi-ideal of S , by theorem (3.7) we have

$$\chi_{F(e)B} \tilde{\cap} S \tilde{\cap} \chi_{F(e)B} = \chi_{F(e)(B \Gamma S \Gamma B)} = \chi_{F(e)B}, \text{ hence } \chi_{F(e)B} \text{ is a fuzzy soft } \Gamma\text{ – bi-ideal of } S.$$

Conversely assume that $\chi_{F(e)B}$ is a fuzzy soft Γ – bi-ideal of S , $B \subseteq S$, by theorem (3.8), it is clear that B is a soft subsemigroup of S .

Let $p \in S$ such that $p \in B \Gamma S \Gamma B$, then $\chi_{F(e)B}$ is a fuzzy soft Γ – bi-ideal, we have $\chi_{F(e)B}(p) = (\chi_{F(e)B} \tilde{\cap} S \tilde{\cap} \chi_{F(e)B})(p) = \chi_{F(e)(B \Gamma S \Gamma B)}(p) = 1$ which implies that $p \in B$ and hence $B \Gamma S \Gamma B \subseteq B$. Hence (F, B) is a soft Γ – bi-ideal of S .

Theorem 3.10: Let (F, A) be a soft Γ – ideal of S if and only if $\chi_{F(e)A}$ (characteristic function) is a fuzzy soft Γ – ideal of S .

Proof: The proof is straightforward.

The following theorem is relation between soft set and fuzzy set.

Theorem 3.11: Let (Q, A) be a soft subset of a Γ – semigroup S , then (Q, A) is a soft Γ – quasi ideal of S if and only if χ_Q (characteristic function) is fuzzy soft Γ – quasi ideal of S .

Proof: Suppose (Q, A) is a soft Γ – quasi ideal of S and $\chi_{F(e)}$ be the characteristic function of S , let $x \in S$. If $x \in (Q, A)$ then $((\chi_Q \tilde{\cap} \chi_{F(e)}) \cap (\chi_{F(e)} \tilde{\cap} \chi_Q))(x) \leq 1 = f_Q(x)$. If $x \notin (Q, A)$ then $x \notin (Q, A) \Gamma (S, E) \cap (S, E) \Gamma (Q, A) \subseteq (Q, A)$.

Case(i): Let $x \notin (Q, A) \Gamma (S, E)$, $x \in (S, E) \Gamma (Q, A)$. If $x = a\gamma_1 b$ then $a \notin (Q, A)$. Then

$$((\chi_Q \circledast \chi_{F(e)}) \cap (\chi_{F(e)} \circledast \chi_Q))(x) = \min\left\{\sup_{x=a\gamma_1 b} \{\min\{\chi_Q(a), \chi_{F(e)}(b)\}\}, \sup_{x=u\gamma_2 v} \{\min\{\chi_{F(e)}(u), \chi_Q(v)\}\}\right\} \\ = 0 = \chi_Q(x).$$

Therefore $(\chi_Q \circledast \chi_{F(e)}) \cap (\chi_{F(e)} \circledast \chi_Q) \subseteq \chi_Q$.

Case (ii): Let $x \in (Q, A)\Gamma(S, E)$, $x \notin (S, E)\Gamma(Q, A)$. If $x = u\gamma_2 v$, then $v \notin (Q, A)$. Then

$$((\chi_Q \circledast \chi_{F(e)}) \cap (\chi_{F(e)} \circledast \chi_Q))(x) = \min\left\{\sup_{x=a\gamma_1 b} \{\min\{\chi_Q(a), \chi_{F(e)}(b)\}\}, \sup_{x=u\gamma_2 v} \{\min\{\chi_{F(e)}(u), \chi_Q(v)\}\}\right\} \\ = 0 = \chi_Q(x).$$

Therefore $(\chi_Q \circledast \chi_{F(e)}) \cap (\chi_{F(e)} \circledast \chi_Q) \subseteq \chi_Q$.

Case (iii): Let $x \notin (Q, A)\Gamma(S, E)$, $x \notin (S, E)\Gamma(Q, A)$. If $x = a\gamma_1 b$ then $a \notin (Q, A)$, then and if $x = u\gamma_2 v$, then $v \notin (Q, A)$. Then

$$((\chi_Q \circledast \chi_{F(e)}) \cap (\chi_{F(e)} \circledast \chi_Q))(x) = \min\left\{\sup_{x=a\gamma_1 b} \{\min\{\chi_Q(a), \chi_{F(e)}(b)\}\}, \sup_{x=u\gamma_2 v} \{\min\{\chi_{F(e)}(u), \chi_Q(v)\}\}\right\} \\ = 0 = \chi_Q(x).$$

Therefore $(\chi_Q \circledast \chi_{F(e)}) \cap (\chi_{F(e)} \circledast \chi_Q) \subseteq \chi_Q$. Hence χ_Q is a fuzzy soft Γ -quasi ideal of S.

Conversely, suppose that χ_Q is a fuzzy soft Γ -quasi ideal of S.

Let $x \in (\chi_Q \circledast \chi_{F(e)}) \cap (\chi_{F(e)} \circledast \chi_Q)$ then there exists $s, t \in S$, $y, z \in (Q, A)$ and $\alpha, \beta \in \Gamma$ such that $x = y\alpha s = t\beta z$.

Consider

$$(\chi_Q \circledast \chi_{F(e)})(x) = \sup_{x=a\gamma_1 b} \{\min\{\chi_Q(a), \chi_{F(e)}(b)\}\} \\ \geq \min\{\chi_Q(y), \chi_{F(e)}(s)\} \\ = \min\{1, 1\} \\ = 1$$

Similarly, $(\chi_{F(e)} \circledast \chi_Q)(x) = 1$. Since $(\chi_Q \circledast \chi_{F(e)}) \cap (\chi_{F(e)} \circledast \chi_Q) \subseteq \chi_Q$.

Consider

$$\chi_Q(x) \geq ((\chi_Q \circledast \chi_{F(e)}) \cap (\chi_{F(e)} \circledast \chi_Q))(x) \\ = \min\{(\chi_Q \circledast \chi_{F(e)})(x), (\chi_{F(e)} \circledast \chi_Q)(x)\} \\ = \min\{1, 1\} = 1$$

Thus $x \in (Q, A)$ and hence $(Q, A)\Gamma(S, E) \cap (S, E)\Gamma(Q, A) \subseteq (Q, A)$. Therefore (Q, A) is a soft Γ -quasi ideal of S.

Theorem 3.12: The following conditions are equivalent

(i) Every soft Γ -bi-ideal is a soft Γ -ideal of S.

(ii) Every fuzzy soft Γ -bi-ideal of S is a fuzzy soft Γ -ideal of S.

Proof: Assume that condition (i) holds, let $\mu_{F(e)}$ be any fuzzy soft Γ -ideal of S. Let $\mu_{F(e)}$ be any fuzzy soft Γ -bi-ideal of S, and $p, q \in S$, since the set $(F, A) \circledast (S, E) \circledast (F, A)$ is a soft Γ -bi-ideal of S, by the assumption is soft Γ -right ideal of S is soft Γ -regular, we have $p\Gamma q \in (p\alpha F(\alpha, \beta)\beta p)\Gamma q \subseteq p\alpha F(\alpha, \beta)\beta p$, there exists $x \in S$ such that $p\Gamma q = p\alpha x\beta p$, since $\mu_{F(e)}$ is a fuzzy soft Γ -bi-ideal of S, $\forall e \in A$.

Consider

$$\begin{aligned}\mu_{F(e)}(p\Gamma q) &= \mu_{F(e)}(p\alpha\beta p) \\ &\geq \min(\mu_{F(e)}(p), \mu_{F(e)}(p)) \\ &= \mu_{F(e)}(p).\end{aligned}$$

Hence $\mu_{F(e)}$ is a fuzzy soft Γ -right ideal of S . Similarly $\mu_{F(e)}$ is a fuzzy soft Γ -left ideal of S . Therefore $\mu_{F(e)}$ is a fuzzy soft Γ -ideal of S . Hence (i) $\Rightarrow$ (ii).

Conversely assume that (ii) holds. Let (F, A) be a soft Γ -ideal of S , by theorem (3.9), the characteristic function $\chi_{F(e)B}$ is a fuzzy soft Γ -bi-ideal of S . Hence by assumption $\chi_{F(e)A}$ is a fuzzy soft Γ -ideal of S , thus by theorem (3.10), $\chi_{F(e)A}$ is soft Γ -ideal of S . Hence (ii) $\Rightarrow$ (i)

The following examples shows that fuzzy soft Γ -ideal and fuzzy soft Γ -bi-ideal of S .

Examples 3.13: Let $S = \{a_1, a_2, a_3, a_4\}$ and $\Gamma = \{\alpha, \beta\}$ in the table.1.

Let $E = \{a_1, a_2, a_3, a_4\}$, $A = \{u_1, u_3\}$ then (F, A) is a fuzzy soft set defined as,

$$\mu_{F(u_1)} = \{(a_1, 0.9), (a_2, 0.3), (a_3, 0.7), (a_4, 0.3)\}$$

$$\mu_{F(u_3)} = \{(a_1, 0.8), (a_2, 0.1), (a_3, 0.5), (a_4, 0.1)\}$$

Hence (F, A) is a fuzzy soft Γ -bi-ideal and fuzzy soft Γ -ideal over S .

Theorem 3.14: The following conditions are equivalent.

(i) (F, A) is a fuzzy soft Γ -ideal of S .

(ii) (F, A) is a fuzzy soft Γ -interior ideal of S .

Proof: Let $\mu_{F(e)}$ is a fuzzy soft Γ -ideal of S , $\forall e \in A$.

We have $\mu_{F(e)}(p\alpha q\beta r) \geq \mu_{F(e)}(q\beta r)$, since $\mu_{F(e)}$ is a Γ -left ideal of S .

$$\geq \mu_{F(e)}(q), \text{ since } \mu_{F(e)} \text{ is a } \Gamma\text{-right ideal of } S.$$

Hence $\mu_{F(e)}(p\alpha q\beta r) \geq \mu_{F(e)}(q) \forall p, q, r \in S$ and $\alpha, \beta \in \Gamma$.

Conversely assume that $\mu_{F(e)}$ is a fuzzy soft Γ -interior ideal of S . Let $p, q \in S$, since S is a soft Γ -regular semigroups, there exists $x, y \in S$, such that $p = p\alpha\beta p$ and $q = q\alpha\beta q$ and $\alpha, \beta \in \Gamma$. Thus we have

$$\begin{aligned}\mu_{F(e)}(p\Gamma q) &= \mu_{F(e)}((p\alpha\beta p)\Gamma q) \\ &= \mu_{F(e)}((p\alpha\beta)\beta p\Gamma q) \\ &\geq \mu_{F(e)}(p)\end{aligned} \quad \text{and}$$

$$\begin{aligned}\mu_{F(e)}(p\Gamma q) &= \mu_{F(e)}(p\Gamma(q\alpha\beta q)) \\ &= \mu_{F(e)}(p\Gamma q\alpha(y\beta q)) \\ &\geq \mu_{F(e)}(q)\end{aligned}$$

Hence proved.

The following example shows that fuzzy soft Γ -ideal and Γ -interior ideal of S .

Examples 3.15: Let $S = \{a_1, a_2, a_3, a_4\}$ and $\Gamma = \{\alpha, \beta\}$ in the table.1.

Let $E = \{a_1, a_2, a_3, a_4\}$, $A = \{u_1, u_3\}$ then (F, A) is a fuzzy soft set defined as,

$$\mu_{F(u_1)} = \{(a_1, 0.8), (a_2, 0.2), (a_3, 0.4), (a_4, 0.2)\}$$

$$\mu_{F(u_3)} = \{(a_1, 0.7), (a_2, 0.3), (a_3, 0.6), (a_4, 0.3)\}$$

Hence (F, A) is a fuzzy soft Γ -interior ideal and fuzzy soft Γ -ideal over S .

Theorem 3.16: Every fuzzy soft Γ -quasi ideals are fuzzy soft Γ -bi-ideal of S .

Proof: Let $\mu_{F(e)}$ is a fuzzy soft Γ - quasi ideal of S . It is sufficient to prove that

$$\mu_{F(e)}(p\alpha q\beta r) \geq \min\{\mu_{F(e)}(p), \mu_{F(e)}(r)\} \quad \forall p, q, r \in S, e \in A \text{ and } \alpha, \beta \in \Gamma.$$

since $\mu_{F(e)}$ is a fuzzy soft Γ -quasi ideal of S . Consider

$$\begin{aligned} \mu_{F(e)}(p\alpha q\beta r) &\geq ((\mu_{F(e)} \tilde{\circ} \chi_{F(e)}) \cap (\chi_{F(e)} \tilde{\circ} \mu_{F(e)}))(p\alpha q\beta r) \\ &= \min\{(\mu_{F(e)} \tilde{\circ} \chi_{F(e)})(p\alpha q\beta r), (\chi_{F(e)} \tilde{\circ} \mu_{F(e)})(p\alpha q\beta r)\} \end{aligned}$$

$$\begin{aligned} (\mu_{F(e)} \tilde{\circ} \chi_{F(e)})(p\alpha q\beta r) &= \sup_{p\alpha q\beta r = u\Gamma v} \min\{\mu_{F(e)}(u), \chi_{F(e)}(v)\} \\ &= \min\{\mu_{F(e)}(p), \chi_{F(e)}(q\beta r)\} \\ &= \mu_{F(e)}(p) \end{aligned}$$

$$\begin{aligned} (\chi_{F(e)} \tilde{\circ} \mu_{F(e)})(p\alpha q\beta r) &= \sup_{p\alpha q\beta r = u\Gamma v} \min\{\chi_{F(e)}(u), \mu_{F(e)}(v)\} \\ &= \min\{\chi_{F(e)}(p\alpha q), \mu_{F(e)}(r)\} \\ &= \mu_{F(e)}(r) \end{aligned}$$

Hence $\mu_{F(e)}(p\alpha q\beta r) \geq \min\{\mu_{F(e)}(p), \mu_{F(e)}(r)\}$.

Theorem 3.17: In a soft Γ -regular semigroup S , then fuzzy soft Γ -quasi-ideals and fuzzy soft Γ -bi-ideals are coincide.

Proof: It is remains to prove that every fuzzy soft Γ -bi-ideals are fuzzy soft Γ -quasi-ideals if $\mu_{F(e)}$ be a fuzzy soft Γ -bi-ideal of S , then

$$(\mu_{F(e)} \tilde{\circ} \chi_{F(e)}) \cap (\chi_{F(e)} \tilde{\circ} \mu_{F(e)}) \subseteq \mu_{F(e)} \dots \dots \dots (i).$$

Let $p \in S$, suppose that $(\mu_{F(e)} \tilde{\circ} \chi_{F(e)})(p) \leq \mu_{F(e)}(p)$, $\forall e \in A$. Consider

$$\begin{aligned} (\mu_{F(e)}(p) \geq (\mu_{F(e)} \tilde{\circ} \chi_{F(e)})(p) \\ \geq \min\{(\mu_{F(e)} \tilde{\circ} \chi_{F(e)})(p), (\chi_{F(e)} \tilde{\circ} \mu_{F(e)})(p)\} \\ = ((\mu_{F(e)} \tilde{\circ} \chi_{F(e)}) \cap (\chi_{F(e)} \tilde{\circ} \mu_{F(e)}))(p) \end{aligned}$$

Now suppose that $(\mu_{F(e)} \tilde{\circ} \chi_{F(e)})(p) \geq \mu_{F(e)}(p)$, then there exists $x, y \in S$, $p = x\Gamma y$ such that $\min\{(\mu_{F(e)}(x) \tilde{\circ} \chi_{F(e)}(y)) \geq \mu_{F(e)}(p)$ that is $\mu_{F(e)}(x) \geq \mu_{F(e)}(p)$(ii), now we prove that $(\chi_{F(e)} \tilde{\circ} \mu_{F(e)})(p) \leq \mu_{F(e)}(p)$, so equation (i) is satisfied. Since

$$(\chi_{F(e)} \tilde{\circ} \mu_{F(e)})(p) = \sup_{p=q\Gamma r} \min\{\chi_{F(e)}(q), \mu_{F(e)}(r)\}.$$

Prove that $\min\{(\chi_{F(e)}(q) \tilde{\circ} \mu_{F(e)}(r)) \leq \mu_{F(e)}(p) \quad \forall p, q, r \in S$, since S is soft Γ -regular, there exists $u \in S$ such that $p = p\alpha u\beta p = x\Gamma y\alpha u\beta q\Gamma r$. Then since $\mu_{F(e)}$ is a fuzzy soft Γ - bi-ideal, we have $\mu_{F(e)}(p) = \mu_{F(e)}(x\Gamma y\alpha u\beta q\Gamma r) \geq \min\{\mu_{F(e)}(x), \mu_{F(e)}(r)\}$.

If $\min\{\mu_{F(e)}(x), \mu_{F(e)}(r)\} = \mu_{F(e)}(x)$, then $\mu_{F(e)}(x) \leq \mu_{F(e)}(p)$, which is impossible by equation (ii), thus $\min\{\mu_{F(e)}(x), \mu_{F(e)}(r)\} = \mu_{F(e)}(r)$, then

$$\mu_{F(e)}(p) \geq \mu_{F(e)}(r) = \min\{\chi_{F(e)}(q), \mu_{F(e)}(r)\}, \forall e \in A.$$

Theorem 3.18: The following conditions are equivalent

(i) (F, A) is Γ -regular.

(ii) $(B, \lambda_1) \cap_R (L, \lambda_2) \subseteq (B, \lambda_1) \tilde{\circ} (L, \lambda_2)$ for every soft Γ -bi-ideal (B, λ_1) and every soft Γ -left ideal (L, λ_2) of S .

Proof: (i) $\Rightarrow$ (ii) By definition $(B, \lambda_1) \tilde{\circ} (L, \lambda_2) = (K_1, \lambda_1 \cap \lambda_2)$ where K_1 is a function $K_1 : (\lambda_1 \cap \lambda_2) \rightarrow P(F(\alpha, \beta))$, $K_1(\alpha, \beta) = B(\alpha, \beta) \Gamma L(\beta) \quad \forall \alpha, \beta \in \lambda_1 \cap \lambda_2$. Now

$(B, \lambda_1) \cap_R (L, \lambda_2) = (K_2, \lambda_1 \cap_R \lambda_2)$ where K_2 is a function $\lambda_1 \cap_R \lambda_2$ to $P(S)$ such that $K_2(\alpha, \beta) = B(\alpha, \beta) \cap L(\beta) \quad \forall \alpha, \beta \in \lambda_1 \cap \lambda_2$.

we suppose that $p \in B(\alpha, \beta) \cap L(\beta)$, since $p \in F(\alpha, \beta)$ and $F(\alpha, \beta)$ is regular, there exists $q \in S$ such that $p = p\alpha q\beta p$. Now $p \in B(\alpha, \beta)$, $q\beta p \in L(\beta)$, and hence

$p = p\alpha q\beta p \in B(\alpha, \beta) \Gamma L(\beta)$. This shows that $(B, \lambda_1) \cap_R (L, \lambda_2) \subseteq (B, \lambda_1) \tilde{\circ} (L, \lambda_2)$.

(ii) $\Rightarrow$ (i). Suppose that $\lambda_1 = \lambda_2 = F(\alpha, \beta)$ and B is a function from $F(\alpha, \beta)$ to $P(F(\alpha, \beta))$ defined by $B(p) = p\alpha F^1(\alpha, \beta)\beta p$, $\forall p \in F(\alpha, \beta)$ and L is a function from

$F(\alpha, \beta)$ to $PF(\alpha, \beta)$ defined by $L(p) = F^1(\alpha, \beta)\beta p$, $\forall p \in F(\alpha, \beta)$. Then $(B, F(\alpha, \beta))$ is a soft Γ -bi-ideal and $(L, F(\alpha, \beta))$ is a soft Γ -left ideal over $F(\alpha, \beta)$ by hypothesis

$p \in B(p) \cap L(p) = B(p) \Gamma L(p) = p\alpha F^1(\alpha, \beta)\beta p \Gamma F^1(\alpha, \beta)\beta p \subseteq p\alpha F^1(\alpha, \beta)\beta p$.

Therefore (F, A) is soft Γ -regular.

Theorem 3.19: Every fuzzy soft Γ -generalized bi-ideal is a fuzzy soft Γ -bi-ideal of S .

Proof: Let $\mu_{F(e)}$ be any fuzzy soft Γ -generalized bi-ideal of S and let $q \in S$, since S is a soft Γ -regular there exists $x \in S$, such that $q = q\alpha x\beta q$. we have

$$\begin{aligned} \mu_{F(e)}(p\Gamma q) &= \mu_{F(e)}(p\Gamma(q\alpha x\beta p)) \\ &= \mu_{F(e)}(p\Gamma(q\alpha x)\beta q) \\ &\geq \min\{\mu_{F(e)}(p), \mu_{F(e)}(q)\}. \end{aligned}$$

This implies that $\mu_{F(e)}$ is a fuzzy soft Γ -subsemigroup of S and so $\mu_{F(e)}$ is a fuzzy soft Γ -bi-ideal of S .

Theorem 3.20: The following conditions are equivalent

(i) (F, A) is Γ -regular.

(ii) $(B, \lambda_1) \cap_R (L, \lambda_2) \subseteq (B, \lambda_1) \tilde{\circ} (L, \lambda_2)$ for every fuzzy soft Γ -generalized bi-ideal (B, λ_1) and every fuzzy soft Γ -left ideal (L, λ_2) of S .

(iii) $(B, \lambda_1) \cap_R (L, \lambda_2) \subseteq (B, \lambda_1) \tilde{\circ} (L, \lambda_2)$ for every fuzzy soft Γ -bi-ideal (B, λ_1) and every fuzzy soft Γ -left ideal (L, λ_2) of S .

Proof: (i) $\Rightarrow$ (ii). By definition $(B, \lambda_1) \tilde{\circ} (L, \lambda_2) = (N, \lambda_1 \cap \lambda_2)$ where N is a function $N : (\lambda_1 \cap \lambda_2) \rightarrow P(F(\alpha, \beta))$, $N(\alpha, \beta) = L(\alpha) \Gamma B(\alpha, \beta) \quad \forall \alpha, \beta \in (\lambda_1 \cap \lambda_2)$. Let (B, λ_1) be a fuzzy soft generalized Γ -bi-ideal and (L, λ_2) be fuzzy soft Γ -left ideal of S . Since (F, A) is soft Γ -regular, let $a \in F(\alpha, \beta)$ there exists $x \in S$ such that $a = a\alpha x\beta a$. Consider

$$\begin{aligned} (\lambda_1 \tilde{\circ} \lambda_2)(a) &= \sup_{a=p\Gamma q} \min\{\lambda_1(p), \lambda_2(q)\} \\ &\geq \min\{\lambda_1(a), \lambda_2(x\beta a)\} \\ &= \min\{\lambda_1(a), \lambda_2(a)\} \\ &= (\lambda_1(a) \cap \lambda_2(a))(a) \end{aligned}$$

Which implies that $(B, \lambda_1) \cap_R (L, \lambda_2) \subseteq (B, \lambda_1) \tilde{\circ} (L, \lambda_2)$.

(ii) $\Rightarrow$ (iii). By theorem 3.19.

(iii) $\Rightarrow$ (i). let (B, λ_1) be a fuzzy soft Γ -bi-ideal and (L, λ_2) be fuzzy soft Γ -left ideal of S , let $a \in (B, \lambda_1) \cap_R (L, \lambda_2)$, by theorem (3.10) and theorem (3.9), $\chi_{F(e)L}$ is a fuzzy soft Γ -left ideal and $\chi_{F(e)B}$ is a fuzzy soft Γ -bi-ideal of S , and $(B, \lambda_1) \tilde{\circ} (L, \lambda_2) = (N, \lambda_1 \cap \lambda_2)$ where $N : (\lambda_1 \cap \lambda_2) \rightarrow P(F(\alpha, \beta))$ and $N(\alpha, \beta) = L(\alpha) \Gamma B(\alpha, \beta) \forall \alpha, \beta \in (\lambda_1 \cap \lambda_2)$. Then by hypothesis $\chi_{F(e)B} \cap \chi_{F(e)L} \subseteq \chi_{F(e)B} \tilde{\circ} \chi_{F(e)L}$.

we have

$$\begin{aligned} (\chi_{F(e)B} \cap \chi_{F(e)L})(p) &= \min\{\chi_{F(e)B}(p), \chi_{F(e)L}(p)\} \\ &= \min\{1, 1\} = 1. \end{aligned}$$

$\forall a \in (B, \lambda_1) \cap_R (L, \lambda_2)$, since $\chi_{F(e)B} \tilde{\circ} \chi_{F(e)L}$ is a fuzzy soft subset of S , we have

$$(\chi_{F(e)B} \tilde{\circ} \chi_{F(e)L})(a) \leq 1 \quad \forall a \in S.$$

Consider

$$\begin{aligned} (\chi_{F(e)B} \tilde{\circ} \chi_{F(e)L})(a) &= \sup_{a=p\Gamma q} \min\{\chi_{F(e)B}(p), \chi_{F(e)L}(q)\} \\ &\geq \min\{\chi_{F(e)B}(a), \chi_{F(e)L}(a)\} \\ &= \min\{1, 1\} = 1. \end{aligned}$$

Hence $\chi_{F(e)B} \tilde{\circ} \chi_{F(e)L} = 1$ thus $1 = \sup_{a=p\Gamma q} \min\{\chi_{F(e)B}(p), \chi_{F(e)L}(q)\}$, which implies that

$\chi_{F(e)B}(p) = 1$ and $\chi_{F(e)L}(q) = 1$ it follows that $p \in (B, \lambda_1)$, $q \in (L, \lambda_2)$ then

$a = p\Gamma q \in (B, \lambda_1) \Gamma (L, \lambda_2)$. Therefore $(B, \lambda_1) \cap_R (L, \lambda_2) \subseteq (B, \lambda_1) \tilde{\circ} (L, \lambda_2)$, by theorem (3.18), hence (F, A) is soft Γ -regular semigroup.

Theorem 3.21: The following conditions are equivalent

(i) (F, A) is left Γ -regular.

(ii) $(I, \lambda_1) \cap_R (B, \lambda_2) \subseteq (I, \lambda_1) \tilde{\circ} (B, \lambda_2)$ for every fuzzy soft Γ -ideal (I, λ_1) and fuzzy soft Γ -bi-ideal (B, λ_2) of S .

Proof: (i) $\Rightarrow$ (ii) By definition $(I, \lambda_1) \tilde{\circ} (B, \lambda_2) = (K, \lambda_1 \cap \lambda_2)$ where K is a function $K : (\lambda_1 \cap \lambda_2) \rightarrow P(F(\alpha, \beta))$, $K(\alpha) = I(\alpha) \Gamma B(\alpha, \beta)$, $\forall \alpha, \beta \in \lambda_1 \cap \lambda_2$. Let (F, A) be soft Γ -left regular and $a \in F(\alpha, \beta)$, then there exists $p \in S$ and $\alpha, \beta \in \Gamma$ such that $a = p\alpha a\beta a$.

Let (I, λ_1) be fuzzy soft Γ -ideal and (B, λ_2) be fuzzy soft Γ -bi-ideal of S . Consider

$$\begin{aligned} (\lambda_1 \tilde{\circ} \lambda_2)(a) &= \sup_{a=p\Gamma q} \min\{\lambda_1(p), \lambda_2(q)\} \\ &\geq \min\{\lambda_1(p\alpha a), \lambda_2(a)\} \\ &\geq \min\{\lambda_1(a), \lambda_2(a)\} \\ &= (\lambda_1 \cap \lambda_2)(a) \end{aligned}$$

Hence $(I, \lambda_1) \cap_R (B, \lambda_2) \subseteq (I, \lambda_1) \tilde{\circ} (B, \lambda_2)$

(ii) $\Rightarrow$ (i) suppose (I, λ_1) and (B, λ_2) are fuzzy soft Γ -ideal and fuzzy soft Γ -bi-ideal of S such that $(I, \lambda_1) \cap_R (B, \lambda_2) \subseteq (I, \lambda_1) \tilde{\circ} (B, \lambda_2)$. Let I be any soft Γ -ideal and B be any soft Γ -bi-ideal of S , $p \in I \cap B$, by theorem (3.10) $\chi_{F(e)I}$ is fuzzy soft Γ -ideal of S and by theorem (3.9) $\chi_{F(e)B}$ is fuzzy soft Γ -bi-ideal of S . Now by theorem (3.7) we have $\chi_{F(e)I \cap B} = \chi_{F(e)I} \cap \chi_{F(e)B}$ and hence

$$\begin{aligned}
 1 &= (\chi_{F(e)I \cap B})(p) \\
 &= (\chi_{F(e)I} \tilde{\cap} \chi_{F(e)B})(p) \\
 &\leq (\chi_{F(e)I} \tilde{\circ} \chi_{F(e)B})(p) \\
 &= \chi_{F(e)I \cap B}(p) \text{ by theorem (3.7)}
 \end{aligned}$$

It follows that $p \in (I, \lambda_1) \tilde{\circ} (B, \lambda_2)$ and hence $(I, \lambda_1) \cap_R (B, \lambda_2) \subseteq (I, \lambda_1) \tilde{\circ} (B, \lambda_2)$. Hence (F, A) is soft left Γ -regular.

4. Fuzzy Soft Gamma Intra Regular Semigroups:

In this section S denotes the soft Γ -intra regular semigroup.

Definition 4.1: A soft Γ -semigroup (F, A) over a semigroup S is called a soft Γ -intra regular semigroup if for each $\alpha, \beta, \gamma \in A$, $F(\alpha, \beta, \gamma)$ is intra regular.

Example 4.2: $S = \{a_1, a_2, a_3, a_4\}$, and $\Gamma = \{\alpha, \beta, \gamma\}$ where α, β, γ is defined on S with the following cayley table:

α	a_1	a_2	a_3	a_4
a_1	a_1	a_1	a_1	a_1
a_2	a_1	a_2	a_3	a_4
a_3	a_1	a_3	a_3	a_3
a_4	a_1	a_3	a_3	a_3

β	a_1	a_2	a_3	a_4
a_1	a_1	a_1	a_1	a_1
a_2	a_1	a_2	a_3	a_3
a_3	a_1	a_3	a_3	a_3
a_4	a_1	a_2	a_3	a_4

γ	a_1	a_2	a_3	a_4
a_1	a_1	a_1	a_1	a_1
a_2	a_1	a_3	a_3	a_3
a_3	a_1	a_3	a_3	a_3
a_4	a_1	a_2	a_3	a_4

Table-2

Consider $E = \{a_1, a_2, a_3, a_4\}$ and $F(a_1) = \{a_1, a_3\}$, $F(a_2) = \{a_1, a_2, a_3\}$, $F(a_3) = \{a_2, a_3, a_4\}$, $F(a_4) = \{a_1, a_3, a_4\}$ Hence (F, S) is soft Γ -intra regular semigroup.

Theorem 4.3: Let (F, A) and (G, B) be two fuzzy soft Γ -ideal (bi-ideal, interior ideal) over soft Γ -intra regular semigroup S , then $(F, A) \wedge (G, B)$ is fuzzy soft Γ -ideal (bi-ideal, interior ideal) over soft Γ -intra regular semigroup S .

Proof: The proof is Straightforward

Theorem 4.4: Let (F, A) and (G, B) be two fuzzy soft Γ -ideal (bi-ideal, interior ideal) over soft Γ -intra regular semigroup S , then $(F, A) \vee (G, B)$ is fuzzy soft Γ -ideal (bi-ideal, interior ideal) over soft Γ -intra regular semigroup S .

Proof: The proof is Straightforward

Theorem 4.5: The following conditions are equivalent.

(i) (F, A) is a fuzzy soft Γ -ideal of S .

(ii) (F, A) is a fuzzy soft Γ -interior ideal of S .

Proof: Let $\mu_{F(e)}$ is a fuzzy soft Γ -ideal of S , $\forall e \in A$.

We have $\mu_{F(e)}(p\alpha q\beta r) \geq \mu_{F(e)}(q\beta r)$, since $\mu_{F(e)}$ is a Γ -left ideal of S .

$\geq \mu_{F(e)}(q)$, since $\mu_{F(e)}$ is a Γ -right ideal of S .

Hence $\mu_{F(e)}(p\alpha q\beta r) \geq \mu_{F(e)}(q) \forall p, q, r \in S$ and $\alpha, \beta \in \Gamma$.

Conversely assume that $\mu_{F(e)}$ is a fuzzy soft Γ -interior ideal of S . Let $p, q \in S$, since S is a soft Γ -intra regular semigroups, there exists $x, y, u, v \in S$, such that $p = x\alpha p\beta p\gamma$ and

$q = u\alpha q\beta q\gamma$ and $\alpha, \beta, \gamma \in \Gamma$. Thus we have

$$\begin{aligned}\mu_{F(e)}(p\Gamma q) &= \mu_{F(e)}((x\alpha p\beta p\gamma)\Gamma q) \\ &= \mu_{F(e)}((x\alpha p)\beta p\gamma(y\Gamma q)) \\ &\geq \mu_{F(e)}(p) \\ \mu_{F(e)}(p\Gamma q) &= \mu_{F(e)}(p\Gamma(u\alpha q\beta q\gamma)) \\ \text{and} \quad &= \mu_{F(e)}(p\Gamma u)\alpha q\beta(q\gamma) \\ &\geq \mu_{F(e)}(q)\end{aligned}$$

Hence proved.

Theorem 4.6: The following conditions are equivalent (i) (F, A) is Γ -intra regular. (ii) $(L, \lambda_1) \cap_R (R, \lambda_2) \subseteq (L, \lambda_1) \tilde{\circ} (R, \lambda_2)$ for every soft Γ -left ideal (L, λ_1) and every soft Γ -right ideal (R, λ_2) of S .

Proof: (i) $\Rightarrow$ (ii) By definition $(L, \lambda_1) \tilde{\circ} (R, \lambda_2) = (K_1, \lambda_1 \cap \lambda_2)$ where K_1 is a function $K_1 : (\lambda_1 \cap \lambda_2) \rightarrow P(F(\alpha, \beta))$, $K_1(\Gamma) = L(\alpha)\Gamma R(\gamma) \quad \forall \Gamma \in \lambda_1 \cap \lambda_2$. we have

$(L, \lambda_1) \cap_R (R, \lambda_2) = (K_2, \lambda_1 \cap_R \lambda_2)$ where K_2 is a function $\lambda_1 \cap_R \lambda_2$ to $P(S)$ such that $K_2(\alpha, \beta) = L(\alpha)\Gamma R(\gamma) \quad \forall \Gamma \in \lambda_1 \cap \lambda_2$.

we suppose that $a \in L(\alpha) \cap R(\gamma)$, since $a \in F(\Gamma)$ and $F(\Gamma)$ is intra regular, there exists $p, q \in S$ such that $a = p\alpha a\beta a\gamma$. Now $p\alpha a \in L(\alpha)$, $a\gamma q \in R(\gamma)$, and hence

$a = p\alpha a\beta a\gamma \in L(\alpha)\Gamma R(\gamma)$ This shows that $(L, \lambda_1) \cap_R (R, \lambda_2) \subseteq (L, \lambda_1) \tilde{\circ} (R, \lambda_2)$

(ii) $\Rightarrow$ (i). Suppose that $\lambda_1 = \lambda_2 = F(\Gamma)$ and N is a function from $F(\Gamma)$ to $P(F(\Gamma))$

defined by $L(a) = F^1(\Gamma)\alpha a$, $\forall a \in F(\Gamma)$ and R is a function from $F(\Gamma)$ to $PF(\Gamma)$ defined by $R(a) = a\gamma F^1(\Gamma)$, $\forall a \in F(\Gamma)$. Then $(L, F(\Gamma))$ is a soft Γ -left ideal and $(R, F(\Gamma))$ is a soft Γ -right ideal over $F(\Gamma)$ by hypothesis

$$a \in L(a) \cap R(a) = L(a)\Gamma R(a) = F^1(\Gamma)\alpha a\beta a\gamma F^1(\Gamma).$$

Therefore (F, A) is soft Γ -intra regular.

Theorem 4.7: The following conditions are equivalent

(i) (F, A) is left Γ -intra regular.

(ii) $(L, \lambda_1) \cap_R (R, \lambda_2) \subseteq (L, \lambda_1) \tilde{\circ} (R, \lambda_2)$ for every fuzzy soft Γ -left ideal (L, λ_1) and fuzzy soft Γ -right ideal (R, λ_2) of S .

Proof: (i) $\Rightarrow$ (ii) By definition $(L, \lambda_1) \tilde{\circ} (R, \lambda_2) = (M_1, \lambda_1 \cap \lambda_2)$ where M_1 is a function $M_1 : (\lambda_1 \cap \lambda_2) \rightarrow P(F(\Gamma))$, $M_1(\Gamma) = L(\alpha)\Gamma R(\gamma)$, $\forall \Gamma \in \lambda_1 \cap \lambda_2$.

$(L, \lambda_1) \cap_R (R, \lambda_2) = (M_2, \lambda_1 \cap_R \lambda_2)$ where M_2 is a function $(\lambda_1 \cap \lambda_2) \rightarrow P(S)$ such that

$$M_2(\Gamma) = L(\alpha) \cap_R R(\gamma) \quad \forall \Gamma \in (\lambda_1 \cap \lambda_2)$$

Let (F, A) be soft Γ -intra regular and $a \in F(\Gamma)$, then there exists $p, q \in S$ and $\alpha, \beta, \gamma \in \Gamma$ such that $a = p\alpha a\beta a\gamma$. Consider

$$\begin{aligned}(\lambda_1 \tilde{\circ} \lambda_2)(a) &= \sup_{a=p\Gamma q} \min\{\lambda_1(p), \lambda_2(q)\} \\ &\geq \min\{\lambda_1(p\alpha a), \lambda_2(a\gamma q)\} \\ &\geq \min\{\lambda_1(a), \lambda_2(a)\} \\ &= (\lambda_1 \cap \lambda_2)(a)\end{aligned}$$

Hence $(L, \lambda_1) \cap_R (R, \lambda_2) \subseteq (L, \lambda_1) \tilde{\circ} (R, \lambda_2)$

(ii) $\Rightarrow$ (i) suppose (L, λ_1) and (R, λ_2) are fuzzy soft Γ -left ideal and fuzzy soft Γ -

right ideal of S such that $(L, \lambda_1) \cap_R (R, \lambda_2) \subseteq (L, \lambda_1) \tilde{\circ} (R, \lambda_2)$, and $a \in (L, \lambda_1) \cap_R (R, \lambda_2)$, by theorem (3.10) $\chi_{F(e)L}$ is fuzzy soft Γ – left ideal of S and $\chi_{F(e)R}$ is fuzzy soft Γ – right ideal of S . Now by theorem (3.7), we have $\chi_{F(e)L \cap_R R} = \chi_{F(e)L} \tilde{\cap} \chi_{F(e)R}$ and hence

$$\begin{aligned} 1 &= (\chi_{F(e)L \cap_R R})(a) \\ &= (\chi_{F(e)L})(a) \tilde{\cap} (\chi_{F(e)R})(a) \\ &= (\chi_{F(e)L} \tilde{\cap} \chi_{F(e)R})(a) \\ &= (\chi_{F(e)L \cap_R R})(a) \text{ by theorem (3.7)} \end{aligned}$$

It follows that $a \in (L, \lambda_1) \tilde{\cap} (R, \lambda_2)$ and hence $(L, \lambda_1) \cap_R (R, \lambda_2) \subseteq (L, \lambda_1) \tilde{\cap} (R, \lambda_2)$. The above theorem hence (F, A) is left Γ – intra regular.

Acknowledgement: The research of the second author is partially supported by UGC-BSR grant: F.25-1/2014-15(BSR)/7 -254/2009(BSR) dated 20-01-2015 in India.

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