



ROUGH BI-IDEALS OF Γ -NEAR-RINGS

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Abstract:

This paper is an extended work of V. S. Subha [10]. Aim of this manuscript is to introduce the notion of rough bi-ideals of Γ -near-ring and study some of its properties.

Key Words: Γ -near-rings, Congruence relation, Rough Bi- ideals.

1. Introduction:

Γ -near-ring and the ideal theory of Γ -near-ring were introduced by Bh. Sathyanarayanan [7]. For basic terminology in near-ring we refer to Pilz [6] and in Γ -near-ring. T.Tamizh Chelvam and N. Meenakumari [11] discussed bi-ideals in Γ -near-ring.

Pawlak [3-5] introduced the theory of rough sets in 1982. It is an another independent method to deal the vagueness and uncertainty. Pawlak used equivalence class in a set as the building blocks for the construction of lower and upper approximations of a set. Many researchers studied the algebraic approach of rough sets in different algebraic structures such as [1,2,8,9]. Thillaigovindan and Subha [12] introduces rough ideals in near-rings. Aim of this manuscript is to introduce the notion of rough bi-ideals and rough quasi-ideals of Γ -near-ring and study some of its properties.

2. Preliminaries and Congruence Relation:

We first recall some basic concepts for the sake of completeness. Recall from[], that a non empty set N with two binary operations $+$ and $\bullet$ multiplication is called a near-ring, if it satisfies the following axioms.

(i) $(N, +)$ is a group; (ii) $(N, \bullet)$ is a semigroup; (iii) $(n_1 + n_2) \bullet n_3 = n_1 \bullet n_3 + n_2 \bullet n_3$, for all $n_1, n_2, n_3 \in N$.

Definition 2.1: [11] A Γ -near-ring is a triple where $(M, +, \Gamma)$ where

- $(M, +)$ is a group
- Γ is non empty set of binary operators on M such that for $\alpha \in \Gamma$, $(M, +, \alpha)$ is a Γ -near-ring
- $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

In Γ -near-ring, $0\gamma x = 0$ and $(-x)\gamma y = -x\gamma y$, but in general $x\gamma 0 \neq 0$ for some $x \in M$. More precisely the above near-ring is right near-ring. $MN_0 = \{n \in M / n\gamma 0 = 0\}$ is called the zero-symmetric part of M and $M = \{n \in M / n\gamma 0 = n\} = \{n \in M / n\gamma n' = n \text{ for all } n' \in M\}$ is called the constant part of M . M is called zero-symmetric if $M = M_0$ and N is called constant if $M = M_c$.

Definition 2.2: [11] A subset I of a Γ -near-ring M is called a left (resp. right) ideal of M , if

- $(I, +)$ is a normal divisor of $(M, +)$ and
- (resp. $x\gamma b \in I$) for all $x \in I, \alpha \in \Gamma$ and $a, b \in M$.

Let I be an ideal of M and X be a non-empty subset of M . Then the sets $\underline{\rho}_I(X) = \{x \in M / x + I \subseteq X\}$ and $\overline{\rho}_I(X) = \{x \in M / (x + I) \cap X \neq \emptyset\}$ are called respectively the lower and upper approximations of the set X with respect to the ideal I .

For any ideal I of M and $a, b \in M$, we say a is congruent to $b \pmod I$, written as $a \equiv b \pmod A$ if $a - b \in I$.

It is easy to see that relation $a \equiv b \pmod A$ is an equivalence relation. Therefore, when $U = M$ and θ is the above equivalence relation, we use the air (M, A) instead of the approximation space (U, θ) .

Also, in this case we use the symbols $\underline{\rho}_I(X)$ and $\overline{\rho}_I(X)$ instead of $\underline{\rho}(X)$ and $\overline{\rho}(X)$. If X is a subset of M , then X^c will be denoted by $M - X$.

Lemma 2.3: [10]

For every approximation space (M, I) and every subsets $X, Y \subseteq M$, the following hold:

- $\underline{\rho}_I(M - Y) = M - \overline{\rho}_I(Y)$
- $\overline{\rho}_I(M - Y) = M - \underline{\rho}_I(Y)$
- $\overline{\rho}_I(X) = (\underline{\rho}_I(X^c))^c$
- $\underline{\rho}_I(X) = (\overline{\rho}_I(X^c))^c$.

Theorem 2.4: [10]

For every approximation space (M, I) and ever subsets $X, Y \subseteq M$, then the following hold:

- $\underline{\rho}_I(X) \subseteq X \subseteq \overline{\rho}_I(X)$

- $\underline{\rho}_I(\emptyset) = \emptyset = \overline{\rho}_I(\emptyset)$
- $\underline{\rho}_I(M) \subseteq M \subseteq \overline{\rho}_I(M)$
- $\overline{\rho}_I(X \cup Y) = \overline{\rho}_I(X) \cup \overline{\rho}_I(Y)$
- $\underline{\rho}_I(X \cap Y) = \underline{\rho}_I(X) \cap \underline{\rho}_I(Y)$
- If $X \subseteq Y$, then $\underline{\rho}_I(X) \subseteq \underline{\rho}_I(Y)$ and $\overline{\rho}_I(X) \subseteq \overline{\rho}_I(Y)$
- $\overline{\rho}_I(X \cap Y) \subseteq \overline{\rho}_I(X) \cap \overline{\rho}_I(Y)$
- $\underline{\rho}_I(X \cup Y) \supseteq \underline{\rho}_I(X) \cap \underline{\rho}_I(Y)$
- If J is an ideal of M such that $I \subseteq J$, then $\underline{\rho}_I(A) \supseteq \underline{\rho}_J(A)$ and $\overline{\rho}_I(A) \subseteq \overline{\rho}_J(A)$
- $\underline{\rho}_I(\underline{\rho}_I(X)) = \underline{\rho}_I(X)$
- $\overline{\rho}_I(\overline{\rho}_I(M)) = \overline{\rho}_I(X)$
- $\overline{\rho}_I(\underline{\rho}_I(M)) = \underline{\rho}_I(X)$
- $\underline{\rho}_I(\overline{\rho}_I(M)) = \overline{\rho}_I(X)$
- $\underline{\rho}_I(x + I) = \overline{\rho}_I(x + I)$ for all $x \in M$.

Corollary 2.5: [10] Let (M, I) be any approximation space. Then

- For every $A \subseteq M$, $\underline{\rho}_I(A)$ and $\overline{\rho}_I(A)$ are definable sets
- For every $x \in M$, $x + I$ is definable set.

Theorem 2.6: [10]

Let I be an ideal of M and A, B nonempty subsets of M , then $\overline{\rho}_I(A) \cap \overline{\rho}_I(B) = \overline{\rho}_I(A \cap B)$.

Theorem 2.7: [10]

Let I be an ideal of $\check{M}$ and A, B nonempty subsets of $\check{M}$, then $\underline{\rho}_I(A) \cap \underline{\rho}_I(B) \subseteq \underline{\rho}_I(A \cap B)$.

Theorem 2.8: [10]

Let I be an ideal of M and A, B nonempty subsets of M , then $\underline{\rho}_I(A) \cap \underline{\rho}_I(B) \subseteq \underline{\rho}_I(A \cap B)$.

Lemma 2.9: [10]

Let I, J be two ideals of M and A a nonempty subset of M , then

- $\underline{\rho}_I(A) \cap \underline{\rho}_J(A) \subseteq \underline{\rho}_{I \cap J}(A)$
- $\overline{\rho}_{I \cap J}(A) \subseteq \overline{\rho}_I(A) \cap \overline{\rho}_J(A)$.

Theorem 2.10: [10]

If I and J are two ideals (resp. sub near-rings) of M , then $\overline{\rho}_I(J)$ is an ideal (resp. sub near-ring) of M .

Theorem 2.11: [10]

If I and J are two ideals (resp. sub Γ -near-rings) of $\check{M}$, then $\underline{\rho}_I(J)$ is an ideal (resp. sub Γ -near-ring) of M .

3. Rough Γ -Near-Rings And Ideals:

In this section we introduce the notion of rough Γ -near-rings and rough ideals and study some of their properties.

Definition 3.1: Let I be an ideal of M and $\rho_I(A) = (\underline{\rho}_I(A), \overline{\rho}_I(A))$ is a *rough set* in the approximation space (M, I) . If $\underline{\rho}_I(A)$ and $\overline{\rho}_I(A)$ are ideals (resp. sub Γ -near-rings) of M , then we call $\rho_I(A)$ *rough ideal* (resp. *Γ -near-ring*). Note that a rough sub Γ -near-ring is also called a *rough Γ -near-ring*. Clearly every rough ideal is a rough Γ -near-ring but the converse need not be true in general.

Lemma 3.2:

- Let I, J be two ideals of M , then $\rho_I(I)$ and $\rho_I(J)$ are rough ideals.
- Let I be an ideal and J a sub Γ -near-ring of M , then $\rho_I(J)$ is a rough Γ -near-ring.

Proof:

From Theorem 2.9 and Theorem 2.10, (i) and (ii) are clear.

Theorem 3.3:

Let I, J be two ideals of M and K be a sub Γ -near-ring of M . then

- $\overline{\rho}_I(K) \cap \overline{\rho}_J(K) \subseteq \overline{\rho}_{I+J}(K)$
- $\underline{\rho}_I(K) \cap \underline{\rho}_J(K) = \underline{\rho}_{(I+J)}(K)$.

Theorem 3.4:

Let I, J be two ideals of M and K a sub Γ -near-ring of M . then

- $\underline{\rho}_{(I+J)}(K) = \underline{\rho}_I(K) + \underline{\rho}_J(K)$
- $\overline{\rho}_{I+J}(K) = \overline{\rho}_I(K) + \overline{\rho}_J(K)$.

4. Rough Quasi-Ideals of Γ -Near-Rings:

In this section we introduce the notion of rough quasi-ideals and rough bi-ideals in Γ -near-rings and characterize Γ -near-ring in terms of rough quasi-ideals and rough bi-ideals. For any two subsets A and B of M , $A \Gamma B = \{a\gamma b \mid a \in A, \gamma \in \Gamma, b \in B\}$.

Definition 4.1: A subgroup Q of $(M, +)$ is said to be *quasi-ideal* of M if $(Q\Gamma M) \cap (M\Gamma Q) \cap (M\Gamma * Q) \subseteq Q$. A subgroup B of $(M, +)$ is said to be a *bi-ideal* of M if $(B\Gamma M\Gamma B) \cap (B\Gamma M)\Gamma * B \subseteq B$. Let M be a zero-symmetric Γ -near-ring. A subgroup Q of $(M, +)$ is said to be a *quasi-ideal* of M if $Q\Gamma M \cap M\Gamma Q \subseteq Q$.

A subgroup B of $(M, +)$ is said to be a *bi-ideal* of M if $B\Gamma M\Gamma B \subseteq B$. Hereafter Γ -near-ring M is a *zero-symmetric* Γ -near-ring unless otherwise mentioned.

Definition 4.2: Let I be an ideal of M . A sub Γ -near-ring A of M is called

- an upper rough quasi-ideal of M if $\overline{\rho}_I(A)$ is a quasi-ideal of M , and
- a lower rough quasi-ideal of M if $\underline{\rho}_I(A)$ is a quasi-ideal of M .

Definition 4.3: Let I be an ideal of M . A sub Γ -near-ring A of M is called

- an upper rough bi-ideal of M if $\overline{\rho}_I(A)$ is a bi-ideal of M , and
- a lower rough bi-ideal of M if $\underline{\rho}_I(A)$ is a bi-ideal of M .

Definition 4.4: Let I be an ideal of M . A sub Γ -near-ring A of M is called

- a rough quasi-bi-ideal of M if $\overline{\rho}_I(A)$ and $\underline{\rho}_I(A)$ are quasi-ideals of M , and
- a rough bi-ideal of M if $\overline{\rho}_I(A)$ and $\underline{\rho}_I(A)$ are bi-ideals of M .

Theorem 4.5:

Let I be an ideal of M . If Q is a quasi-ideal of M , then $\overline{\rho}_I(Q)$ is a quasi-ideal of M .

Proof:

Let Q is a quasi-ideal of M . Let $x \in (\overline{\rho}_I(Q)\Gamma M) \cap (M\Gamma \overline{\rho}_I(Q))$. Then there exists $a, d \in \overline{\rho}_I(Q)$ and $b, c \in M$ such that $x = a\gamma b \in \overline{\rho}_I(Q)\Gamma M$ and $x = c\gamma d \in M\Gamma \overline{\rho}_I(Q)$. This means that $(a + I) \cap Q \neq \emptyset$ and $(d + I) \cap Q \neq \emptyset$. This implies that there exists y such that $y \in a + I$ and $y \in Q$. Now $y - a \in I$. Since I is a right ideal and Q is a quasi-ideal of M , $y\gamma b - a\gamma b \in I$ and $y\gamma b \in Q$. This means that $(a\gamma b + I) \cap Q \neq \emptyset$ and $x \in \overline{\rho}_I(Q)$.

Now $(\overline{\rho}_I(Q)\Gamma M) \cap (M\Gamma \overline{\rho}_I(Q)) \subseteq \overline{\rho}_I(Q)\Gamma M \subseteq \overline{\rho}_I(Q)$. Thus $\overline{\rho}_I(Q)$ is a quasi-ideal of M .

Theorem 4.6:

Let I be an ideal of M . If Q is a quasi-ideal of M and $\underline{\rho}_I(Q)$ is nonempty, then $\underline{\rho}_I(Q)$ is a quasi-ideal of M .

Proof:

Since Q is a subgroup of $(M, +)$, $\underline{\rho}_I(Q)$ is a subgroup of M . Let $x \in (\underline{\rho}_I(Q)\Gamma M) \cap (M\Gamma \underline{\rho}_I(Q))$. Then $x \in \underline{\rho}_I(Q)\Gamma M$ and $x \in M\Gamma \underline{\rho}_I(Q)$. This implies that $x = a\gamma n_1 \in \underline{\rho}_I(Q)\Gamma M$ and $x = n_2\gamma b \in M\Gamma \underline{\rho}_I(Q)$. Now $a + I \subseteq Q$, $a\gamma n_1 + I \subseteq Q\gamma n_1$. Thus $x + I \subseteq Q\gamma M$. Again $x = n_2\gamma b \in M\Gamma \underline{\rho}_I(Q)$ implies $b \in \underline{\rho}_I(Q)$. This means that $B + I \subseteq Q$. Since $0 \in I$, $n_2\gamma b + I \subseteq M\Gamma Q$. Combining these results $(x + I) \subseteq (Q\Gamma M) \cap (M\Gamma Q) \subseteq Q$. Thus $x \in \underline{\rho}_I(Q)$.

Hence $(\underline{\rho}_I(Q)\Gamma M) \cap (M\Gamma \underline{\rho}_I(Q)) \subseteq \underline{\rho}_I(Q)$ which implies that $\underline{\rho}_I(Q)$ is quasi-ideal of M .

Theorem 4.7:

Let I be an ideal of M . If Q is a quasi ideal of M and $\underline{\rho}_I(Q)$ is nonempty, then $\rho_I(Q) = (\underline{\rho}_I(Q), \overline{\rho}_I(Q))$ is a quasi-ideal of M .

Proof:

It follows from Theorem 4.5 and Theorem 4.6.

Theorem 4.8:

Let I be an ideal of M . If B is a bi-ideal of M , then $\overline{\rho}_I(B)$ is a bi-ideal of M .

Proof:

Let B be a bi-ideal of M . Since by Theorem 3.3(3), $\overline{\rho}_I(M) = M$,

$$\begin{aligned} \overline{\rho}_I(B)\Gamma M\Gamma \overline{\rho}_I(B) &= \overline{\rho}_I(B\Gamma M\Gamma B), \text{ by Theorem 3.8} \\ &\subseteq \overline{\rho}_I(B). \end{aligned}$$

Thus $\overline{\rho}_I(B)$ is a bi-ideal of M .

Theorem 4.9:

Let I be an ideal of M . If B is a bi-ideal of M and $\underline{\rho}_I(B)$ is nonempty, then $\underline{\rho}_I(B)$ is a bi-ideal of M .

Proof:

Let B be a bi-ideal of M . Since by Theorem 3.3(3), $\varrho_I(M) = M$ and by Theorem 3.8,

$$\begin{aligned}\varrho_I(B)\Gamma M\Gamma \varrho_I(B) &\subseteq \varrho_I(B\Gamma M\Gamma B) \\ &\subseteq \varrho_I(B).\end{aligned}$$

Thus $\varrho_I(B)$ is a bi-ideal of M .

Theorem 4.10:

Let I be an ideal of M . If B is a bi-ideal of M . If B is a bi-ideal of M and $\rho_I(B)$ is nonempty, $\rho_I(B)$ is a rough bi-ideal of M .

Proof:

It is straight forward by Theorem 4.8 and theorem 4.9.

Lemma 4.11:

Let I be an ideal of M . If B is a quasi-ideal of M and $\rho_I(B)$ is nonempty, then

- $\bar{\rho}_I(B)$ is a bi-ideal of M
- $\rho_I(B)$ is a bi-ideal of M .

Proof:

Let B be a quasi-ideal of M .

- By Theorem 5.5, $\bar{\rho}_I(B)$ is a quasi-ideal of M . Now

$$\begin{aligned}\bar{\rho}_I(B)\Gamma M\Gamma \bar{\rho}_I(B) &= \bar{\rho}_I(B\Gamma M\Gamma B) \\ &\subseteq \bar{\rho}_I(B\Gamma M\Gamma M) \\ &\subseteq \bar{\rho}_I(B\Gamma M).\end{aligned}$$

Also

$$\begin{aligned}\bar{\rho}_I(B)\Gamma M\Gamma \bar{\rho}_I(B) &= \bar{\rho}_I(B\Gamma M\Gamma B) \\ &\subseteq \bar{\rho}_I(M\Gamma M\Gamma B) \\ &\subseteq \bar{\rho}_I(M\Gamma B).\end{aligned}$$

Thus

$$\begin{aligned}\bar{\rho}_I(B)M\Gamma M\bar{\rho}_I(B) &= \bar{\rho}_I(B\Gamma M) \cap \bar{\rho}_I(M\Gamma B) \\ &\subseteq (\bar{\rho}_I(B)\Gamma M) \cap (M\Gamma \bar{\rho}_I(B)) \\ &\subseteq \bar{\rho}_I(B).\end{aligned}$$

Hence $\bar{\rho}_I(B)$ is a bi-ideal of M .

- By Theorem 4.6, $\varrho_I(B)$ is a quasi-ideal of M .

Consider

$$\begin{aligned}\varrho_I(B)\Gamma M\Gamma \varrho_I(B) &\subseteq \varrho_I(B\Gamma M\Gamma B) \\ &\subseteq \varrho_I(B\Gamma M\Gamma M) \\ &\subseteq \varrho_I(B\Gamma M).\end{aligned}$$

Again

$$\begin{aligned}\varrho_I(B)\Gamma M\Gamma \varrho_I(B) &\subseteq \varrho_I(B\Gamma M\Gamma B) \\ &\subseteq \varrho_I(M\Gamma M\Gamma B) \\ &\subseteq \varrho_I(M\Gamma B).\end{aligned}$$

$$\begin{aligned}\varrho_I(B)\Gamma M\Gamma \varrho_I(B) &\subseteq \varrho_I(B\Gamma M) \cap \varrho_I(M\Gamma B) \\ &\subseteq \varrho_I(B\Gamma M \cap M\Gamma B) \text{ by Theorem 3.3(5)} \\ &\subseteq \varrho_I(B).\end{aligned}$$

Lemma 4.12:

Let I be any ideal of M . If B is a quasi-ideal of M , then $\rho_I(B)$ is a bi-ideal of M .

Theorem 4.13:

Let I be any ideal of M . If A is a left ideal (right ideal, sub Γ -near-ring, two-sided ideal) of M , then $\bar{\rho}_I(A)$ is a quasi-ideal of M .

Proof:

Let A be any ideal of M . Since M is a zero-symmetric near-ring, $MA \subseteq A$.

Consider $(\bar{\rho}_I(A)\Gamma M) \cap M\Gamma \bar{\rho}_I(A) \subseteq M\Gamma \bar{\rho}_I(A) = \bar{\rho}_I(M\Gamma A) \subseteq \bar{\rho}_I(A)$. Thus

$(\bar{\rho}_I(A)\Gamma M) \cap (M\Gamma \bar{\rho}_I(A)) \subseteq \bar{\rho}_I(A)$. Hence $\bar{\rho}_I(A)$ is a quasi-ideal of M .

Theorem 4.14:

Let I be an ideal of M . If A is a left ideal (right ideal, sub Γ -near-ring, two-sided ideal) of M , then $\bar{\rho}_I(A)$ is a bi-ideal of M .

Proof:

Let A be any ideal of M . By Theorem 4.5, $\overline{\rho}_\Gamma(A)$ is a quasi-ideal of M . By Theorem 4.8, $\overline{\rho}_\Gamma(A)$ is a bi-ideal of M .

Theorem 4.15:

Let I be any ideal of M . If A is a left ideal (right ideal, sub Γ -near-ring, two-sided ideal) of M , then $\rho_I(A)$ is nonempty, then $\rho_I(A)$ a quasi-ideal of M .

Proof:

Let A be any ideal of M . Since M is a zero-symmetric Γ -near-ring, $M\Gamma A \subseteq A$. Consider $\rho_\Gamma(A)\Gamma M \cap M\Gamma \rho_\Gamma(A) \subseteq M\Gamma \rho_\Gamma(A) = \rho_\Gamma(M\Gamma A) \subseteq \rho_\Gamma(A)$. Thus $\rho_\Gamma(A)$ is a quasi-ideal of M .

Theorem 4.16:

Let I be an ideal of M . If A is a left ideal (right ideal, sub Γ -near-ring, two-sided ideal) of M , then $\rho_I(A)$ is nonempty, then $\rho_I(A)$ a bi-ideal of M .

Proof:

Let A be any ideal of M . By Theorem 4.6, $\rho_\Gamma(A)$ is a quasi-ideal of M . By Theorem 4.9, $\rho_\Gamma(A)$ is a bi-ideal of M .

Theorem 4.17:

Let I be an ideal of M . If A is a left ideal (right ideal, sub near-ring, two-sided ideal) of M , then $\rho_I(A)$ is nonempty, then $\rho_I(B)$ a rough bi-ideal of M .

Proof:

The result follows from Theorem 4.14 and 4.16.

5. Conclusion:

The theory of Γ -near ring and theory of rough sets have many applications in various fields. In this paper is to present the concepts of congruence relation in Γ -near-rings and the lower and upper approximations of an ideal with respect to the congruence relation and introduce the notion of rough ideals in Γ -near-rings, which is a generalization of the notion of near rings. Some properties of the lower and upper approximations are discussed in Γ -near-rings. The definitions and results are extended to rings.

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